Implementing nonlinear flame models in Frequency Domain LOTAN to obtain bifurcation diagrams

Giovanni Campa

Hopkinson Laboratory, Department of Engineering, University of Cambridge

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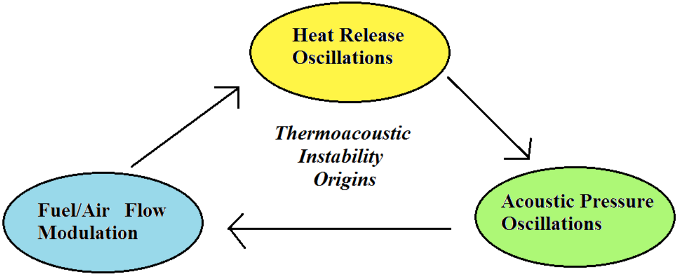
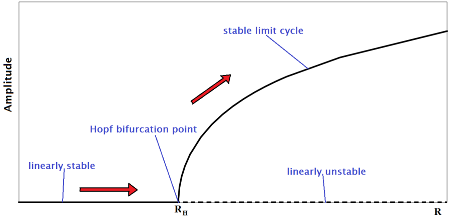
Thermoacoustic Instabilities



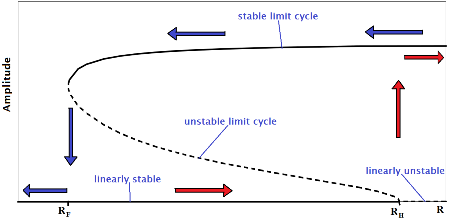
The instability is due to high amplitude pressure oscillations. Such oscillations can lead to violent vibrations within the system, with the risk of a complete failure.

Nonlinear Analysis

Linear flame models are not able to predict triggering instabilities and limit cycle amplitudes $\Longrightarrow$ Nonlinearities are introduced inside heat release rate



Supercritical Bifurcation



Subcritical Bifurcation

Motivation & Objective

$\Longrightarrow$ Bifurcation diagrams can be obtained through:

- Sistematic variation of parameters and tracking direct time integration. This method is computationally expensive;
- Numerical continuation. An unstable limit cycle can be computed, it is very efficient in obtaining the dependance of the solution from the control parameter. It takes a long time to map the bifurcation diagram.
- DDE-BIFTOOL is a sofware based on the numerical continuation methods for delay systems.

$\Longrightarrow$ It could be easy for acoustic network models such as **LOTAN** to map the bifurcation diagram as a function of one control parameter.

LOTAN's approach

- LOTAN is an Acoustic Network model which uses the linear theory for predicting the combustion oscillations in LPP combustors.
- Introducing nonlinear flame models, it is possible to predict the limit cycle amplitudes both in the frequency and in the time domain.

- Linear Flame Model $\Longrightarrow$ $\frac{Q'}{\overline{Q}} = -k\frac{m'}{\overline{m}}$

- Nonlinear Flame Transfer Function $\Longrightarrow$ $TM(\omega, A) = \frac{\widehat{Q}^{(1)}}{\widehat{Q}^{L}} TM^{L}(\omega)$

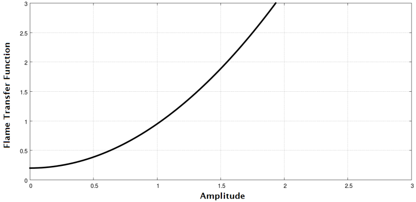
Configuration

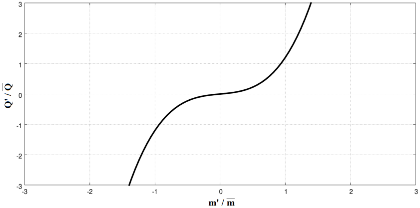
- Rijke Tube;
- Flame located at one quarter of the tube;
- Temperature increases from 300 K to 700 K across the flame;
- Time delay is constant (now it is not important the amplitude dependance of the time delay);
- Open-end inlet and outlet boundary conditions, $p' = 0$.

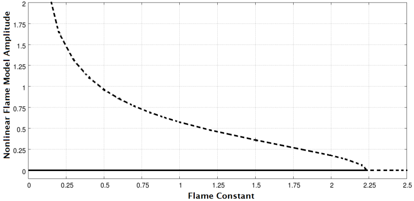
Flame Model 1

$$\frac{Q'}{\overline{Q}} = -k\left[\mu_2\left(\frac{m'}{\overline{m}}\right)^3 + \mu_0\frac{m'}{\overline{m}}\right]$$

$$\mu_2 = 1 \qquad \mu_0 = 0.2$$







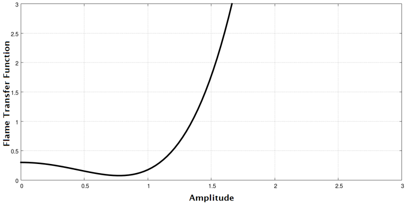
$\Longrightarrow$ Ref.: N. Ananthkrishnan, K. Sudhakar, S. Sudershan and A. Agarwal, *Application of Secondary Bifurcations to Large-Amplitude Limit Cycles in Mechanical Systems*, Journal of Sound and Vibration, (1998) 215 (1), 183-188.

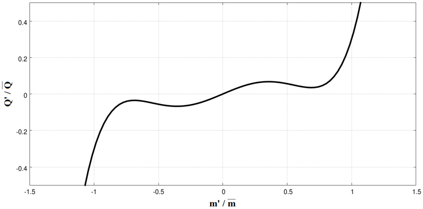
Flame Model 2

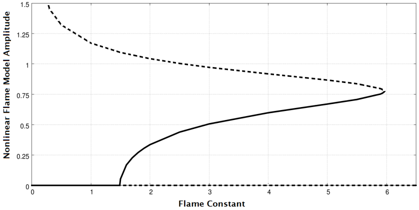
$$\frac{Q'}{\overline{Q}} =$$

$$-k\left[\mu_4\left(\frac{m'}{\overline{m}}\right)^5 + \mu_2\left(\frac{m'}{\overline{m}}\right)^3 + \mu_0\frac{m'}{\overline{m}}\right]$$

$$\mu_4 = 1 \qquad \mu_2 = -1 \qquad \mu_0 = 0.3$$





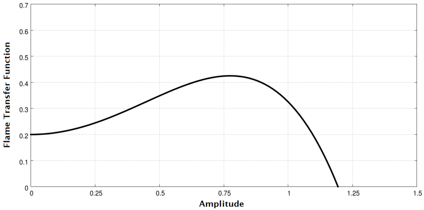


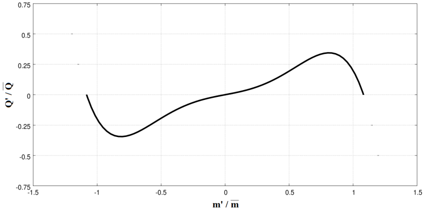
⟹ Ref.: N. Ananthkrishnan, K. Sudhakar, S. Sudershan and A. Agarwal, *Application of Secondary Bifurcations to Large-Amplitude Limit Cycles in Mechanical Systems*, Journal of Sound and Vibration, (1998) 215 (1), 183-188.

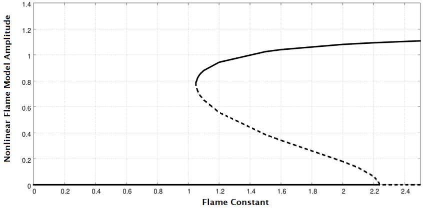
Flame Model 3

$$\frac{Q'}{\overline{Q}} = -k\left[\mu_4\left(\frac{m'}{\overline{m}}\right)^5 + \mu_2\left(\frac{m'}{\overline{m}}\right)^3 + \mu_0\frac{m'}{\overline{m}}\right]$$

$$\mu_4 = -1 \qquad \mu_2 = 1 \qquad \mu_0 = 0.2$$







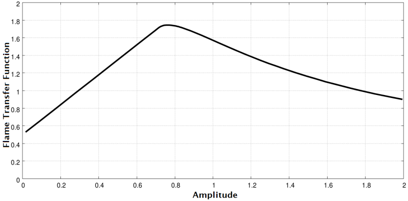
$\Longrightarrow$ Ref.: N. Ananthkrishnan, K. Sudhakar, S. Sudershan and A. Agarwal, *Application of Secondary Bifurcations to Large-Amplitude Limit Cycles in Mechanical Systems*, Journal of Sound and Vibration, (1998) 215 (1), 183-188.

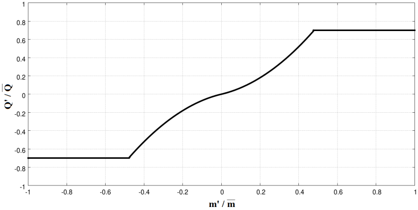
Flame Model 4

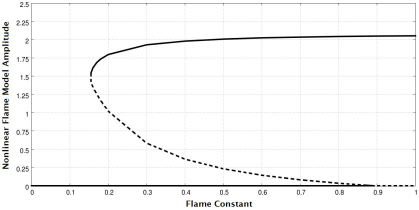
$$\frac{Q'}{\overline{Q}} = -k\left[C\left|\frac{m'}{\overline{m}}\right| + C_1\right]\frac{m'}{\overline{m}}$$

$$Q'(t) = \begin{cases} Q'(t) & \text{for } |Q'(t)| \leq \alpha\overline{Q} \\ \alpha\overline{Q}\,sign(Q'(t)) & \text{for } |Q'(t)| > \alpha\overline{Q} \end{cases}$$

$C = 2 \qquad C_1 = 0.5$



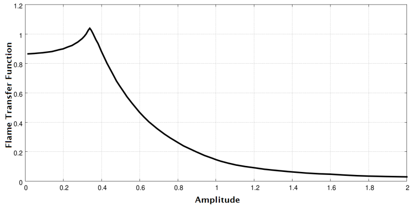


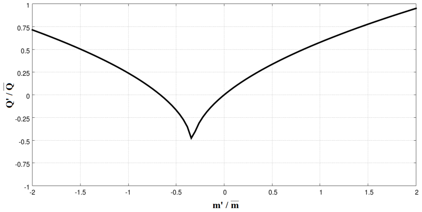


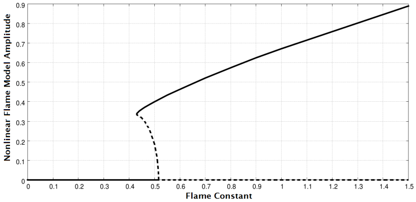
$\Longrightarrow$ Ref.: N. Ananthkrishnan, S. Deo and F.E.C. Culick, *Reduced-Order Modeling and Dynamics of Nonlinear Acoustic Waves in a Combustion Chamber*, Combustion Science and Technology, 177, 1-27,2005.

Flame Model 5

$$\frac{Q'}{\overline{Q}} = -k\left(\sqrt{\left|\frac{1}{3} + \frac{m'}{\overline{m}}\right|} - \sqrt{\frac{1}{3}}\right)$$







$\Longrightarrow$ Ref.: M. Juniper, *Triggering in the Horizontal Rijke Tube: Non-normality, transient growth and bypass transition*, J. Fluid Mech. (2011), vol. 667, pp. 272-308.

Comments

- Flame Transfer Function increases from the value at zero amplitude $\Longrightarrow$ Subcritical Bifurcation;
- Flame Transfer Function decreases from the value at zero amplitude $\Longrightarrow$ Supercritical Bifurcation;

- Flame Model: positive third derivative around zero $\Longrightarrow$ Subcritical Bifurcation;
- Flame Model: negative third derivative around zero $\Longrightarrow$ Supercritical Bifurcation;

- A linear damping coefficient is necessary in order to get the Hopf bifurcation point;
- Saturation permits to have a stable periodic solution, avoiding very large oscillations.

Conclusions

- LOTAN has shown a very good behaviour in mapping the bifurcation diagrams;
- It is very simple to use;
- It does not take a lot of time for the analysis of the system in the frequency domain;

- The results are in a very good agreement with the ones from the literature, which have been obtained through ad hoc codes.