Low order modelling of a partially premixed flames

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  - *Large amplitude pressure oscillations in the combustion chamber, driven by coupling between flames and acoustic waves*
- But why are they of interest?
  - Because they do serious damage
  - Because they affect a wide range of equipment
  - Industrial shift towards lean-burn, low-NO_x combustion systems
    - dramatically increase susceptibility to instability

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  - Experimental tests
  - High-order reacting CFD
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Increasing Accuracy

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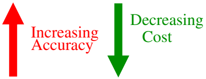

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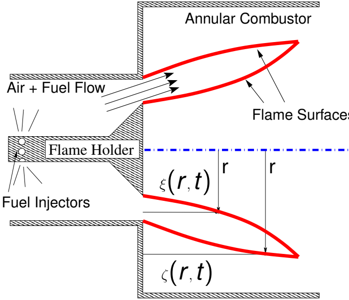
G-Equation Flame Model

- Two thin axisymmetric flame surfaces, evolving kinematically
- Compressibility and vorticity are ignored



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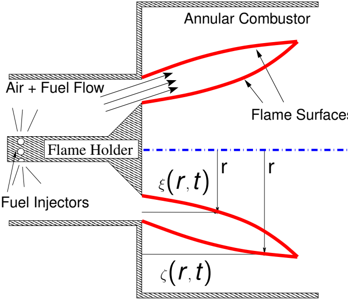

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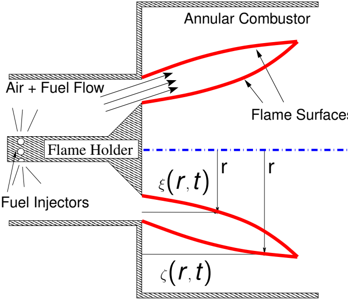

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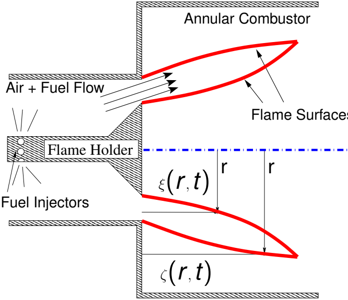

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- Compressibility and vorticity are ignored
- Flame surfaces can pinch off and multiple areas of combustion are possible
- Fluctuation of flame speed: $S_u = f(\phi)$
- Heat release is proportional to the flame area:

$$Q(t) = 2\pi\rho_u\eta\left[\int_a^c S_u(\phi)\Delta H(\phi) r\sqrt{1+\left(\frac{\partial\xi}{\partial r}\right)^2}dr + \int_b^c S_u(\phi)\Delta H(\phi) r\sqrt{1+\left(\frac{\partial\zeta}{\partial r}\right)^2}dr\right]$$

G-Equation Flame Model - Results

- 160Hz, 30% forcing. Equivalence ratio fluctuations dominate flame surface wrinkling

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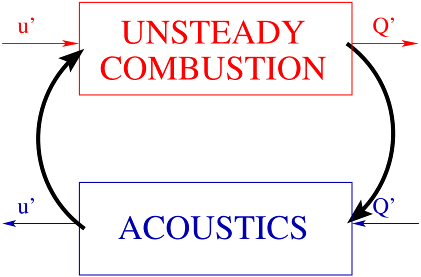

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- The flame transfer function can come:
  - From experiment
  - From CFD
  - Simple descriptions (time-lag) with saturation
  - From low-order flame models

Acoustic Coupling with Flame Model

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  - Describing Function
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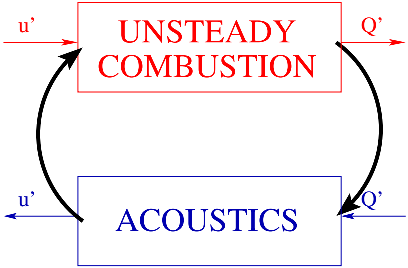
Acoustic Coupling - Frequency Domain

- Fitting a linear TF to the calculated flame response of the form:

$$H_{linear}(i\omega) = \frac{a_n(i\omega)^n + a_{n-1}(i\omega)^{n-1} + \cdots + a_1(i\omega) + a_0}{b_n(i\omega)^n + b_{n-1}(i\omega)^{n-1} + \cdots + b_1(i\omega) + b_0}$$

- Provides unsteady heat release for the acoustic network model



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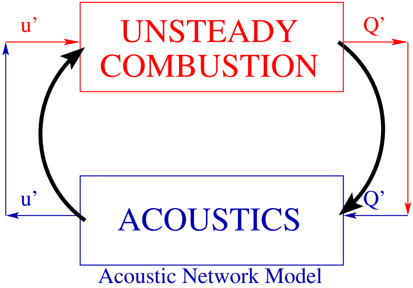
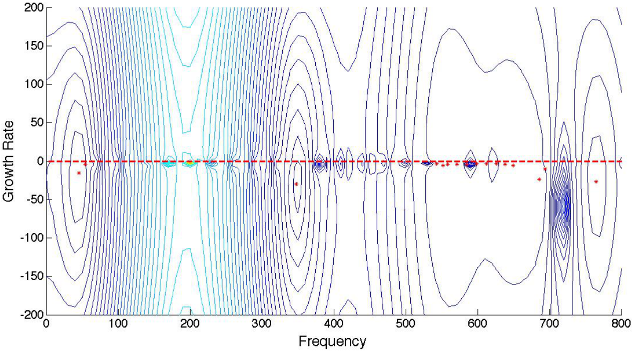


Acoustic Coupling - Frequency Domain

- Plotting error at downstream boundary - a local minimum means we have found a mode
- If the mode's growth rate is negative $\Rightarrow$ stable mode, Growth rate postive $\Rightarrow$ unstable



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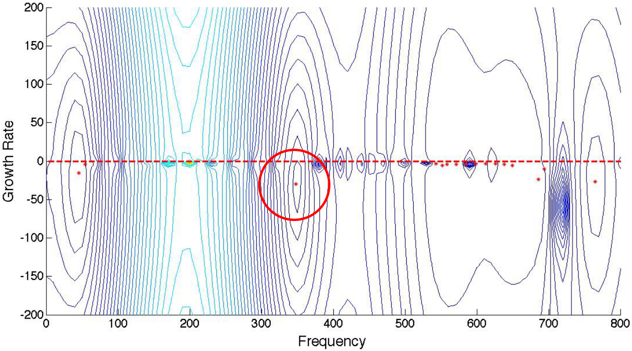

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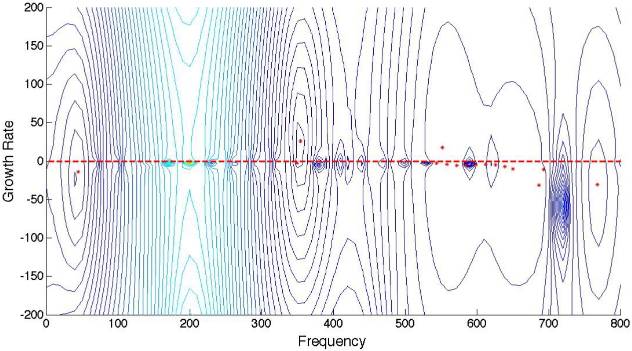

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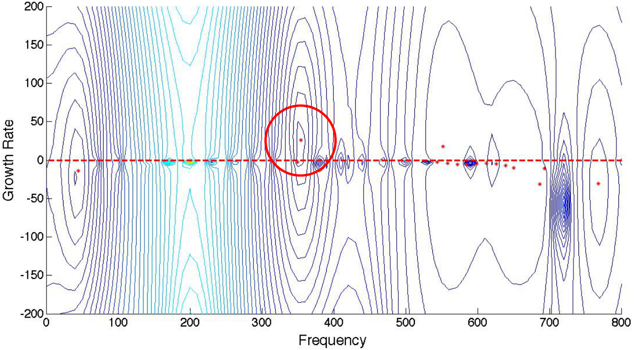

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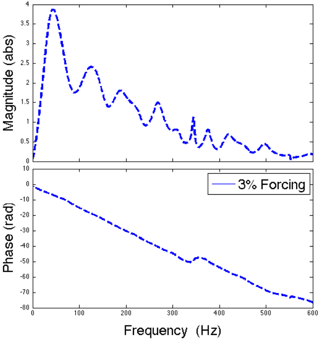
- A Describing Function $K(\omega, A)$ is defined to describe the saturation of the FTF

$$K(\omega, A) = \frac{H_{nonlinear}(\omega)}{H_{linear}(\omega)}$$

- The CLTF is then given by

$$CLTF = \frac{K(\omega, A)H(\omega)}{1 + K(\omega, A)H(\omega)G_{ac}}$$



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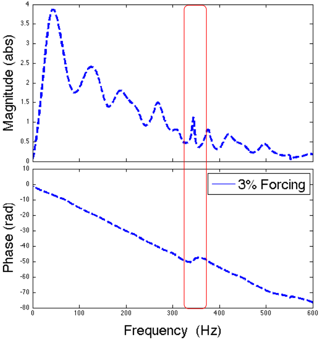

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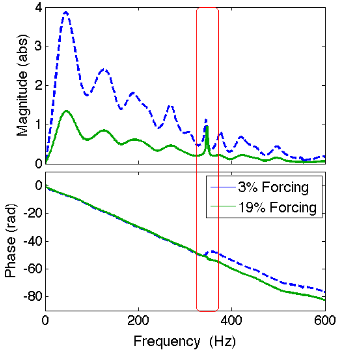
- A is increased until unstable mode at $\sim 350Hz$ is stabilised
- This gives a LCA of 19%, compared with the experimental result of 21%



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- More realistic flame saturation, mode interaction
- Acoustics assumed to remain linear, $G_{ac}(\omega)$ output from LOTAN

$$\frac{u_G'(t)}{\overline{u_G}} = \int G_{ac}(t-\tau)\frac{Q'(\tau)}{\overline{Q}}d\tau$$

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- Model started with low-level broadband noise, and allowed to develop

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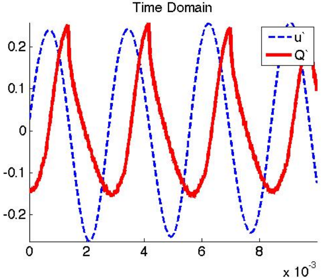

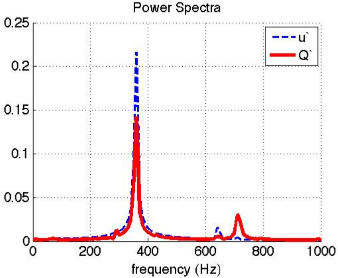

	Experiment	Time Domain Coupling
Frequency	$348Hz$	$359Hz$
Limit Cycle Amplitude	25%	21%

Conclusions

- Thermo-acoustic Instabilities are a problem affecting many real-world systems, leading to large-scale damage and costs
- Low-order flame model for a realistic geometry can capture the essential behaviour of partially-premixed flames
- Coupling with an acoustic model can be achieved in time and frequency domain
- Used this coupling, this approach can provide valuable predictions of the occurrence, frequency, and amplitude of instabilities

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Future Work

- Flame model being extended to incorporate effects of vorticity, flame stretch, etc.
- Technique can also be applied to more realistic modes found in gas turbines