Feedback control of vortex shedding

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Overview

- Introduction
- A model-based control approach
- Control at higher Reynolds numbers
- Explaining previous results

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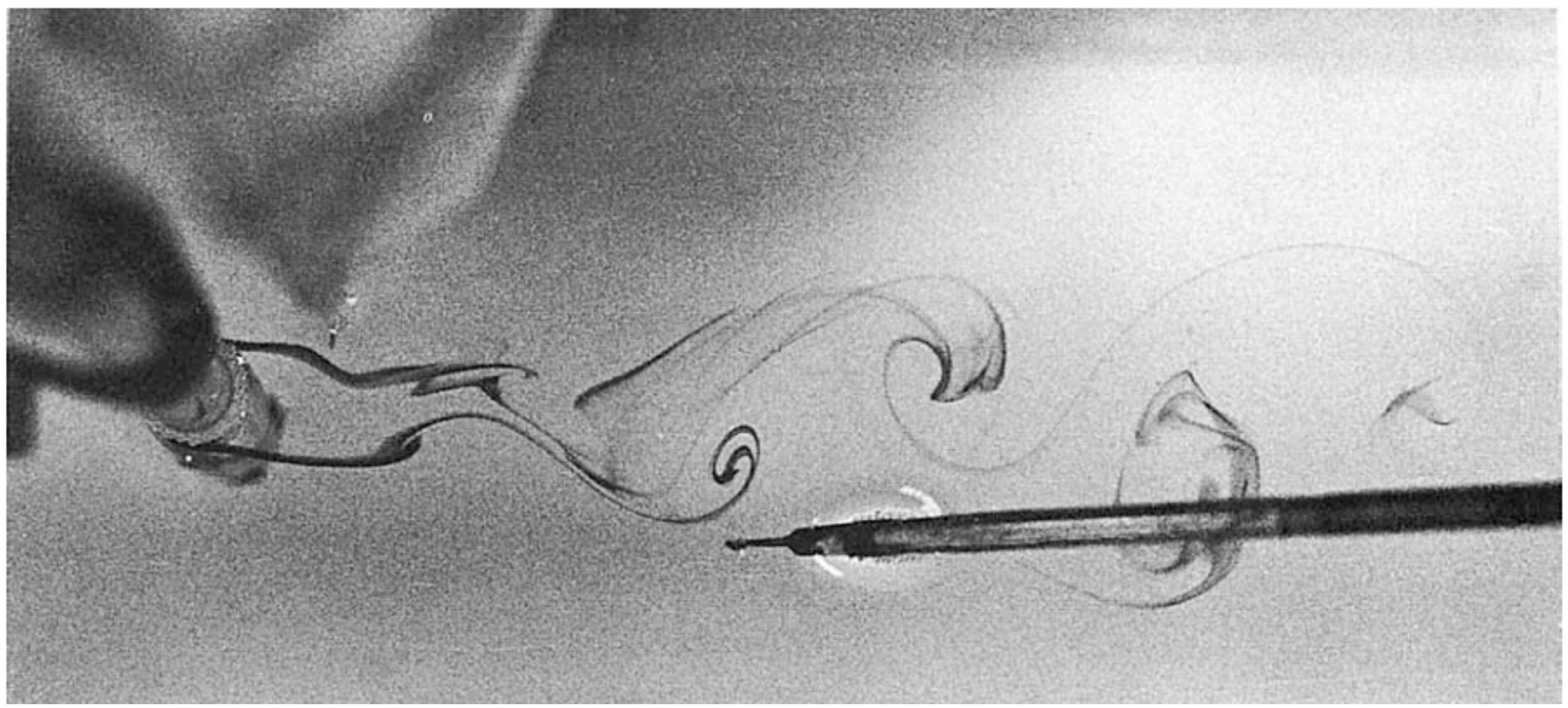
Introduction: the cylinder wake

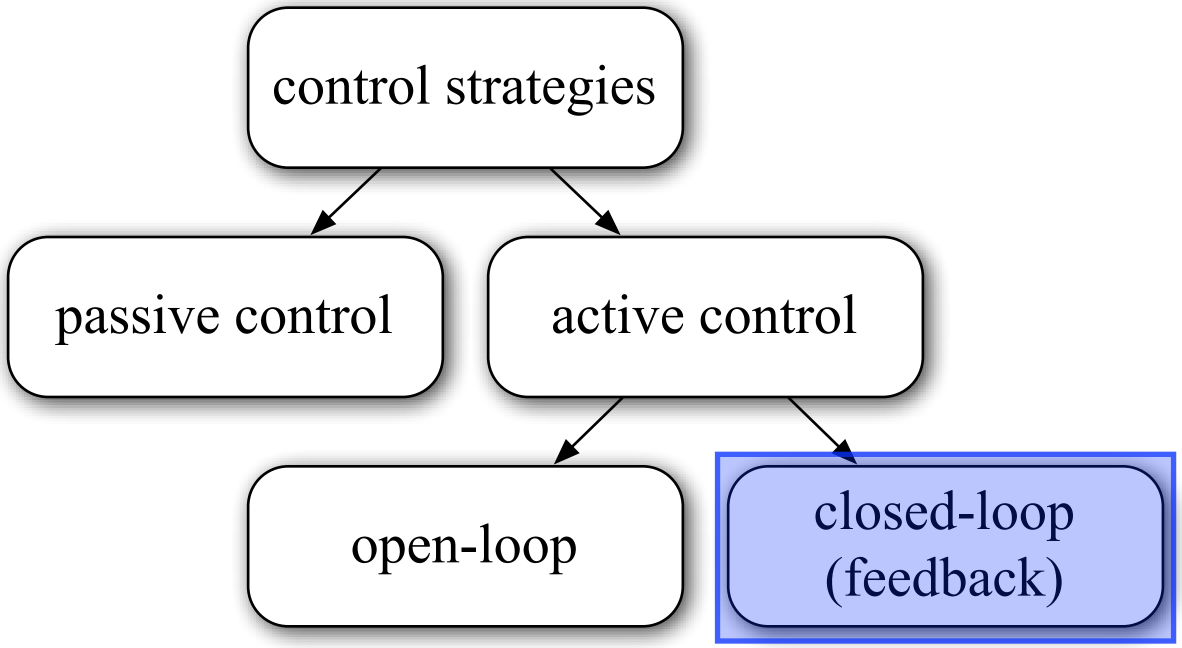
- the onset of the von Karman vortex street occurs at a Reynolds number of about **49** (Williamson, Annu. Rev. Fluid Mech. 1996)

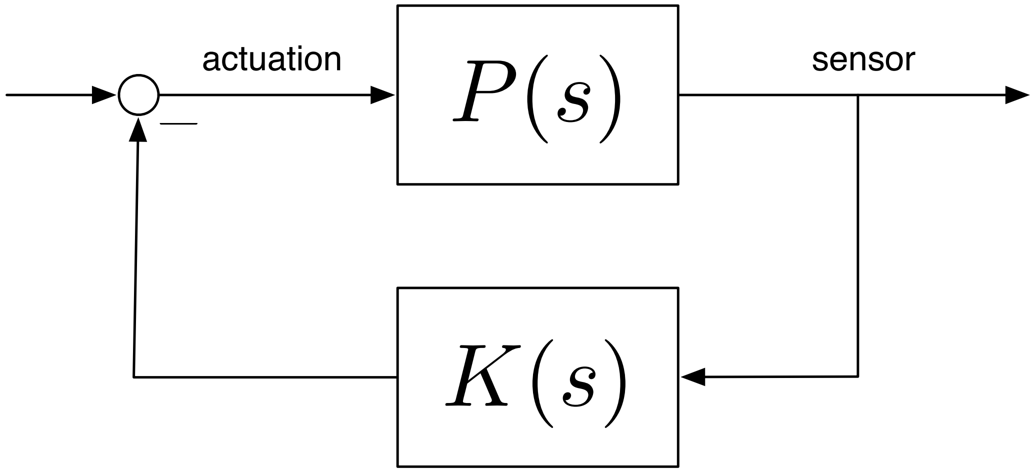


Roussopoulos, JFM 1993
Re=120

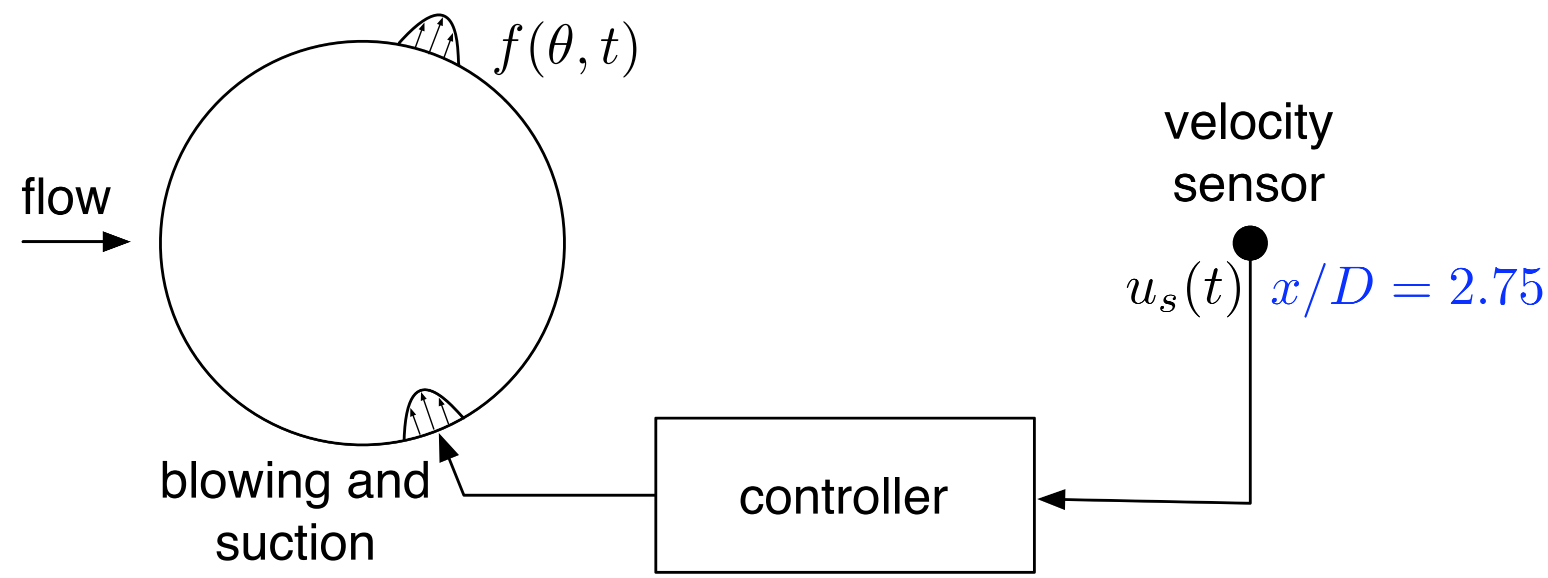
- the vortices lead to increased drag; increased fluctuations in lift; structural vibration; and noise
- we focus our attention on the two-dimensional wake using direct numerical simulations

Introduction: control



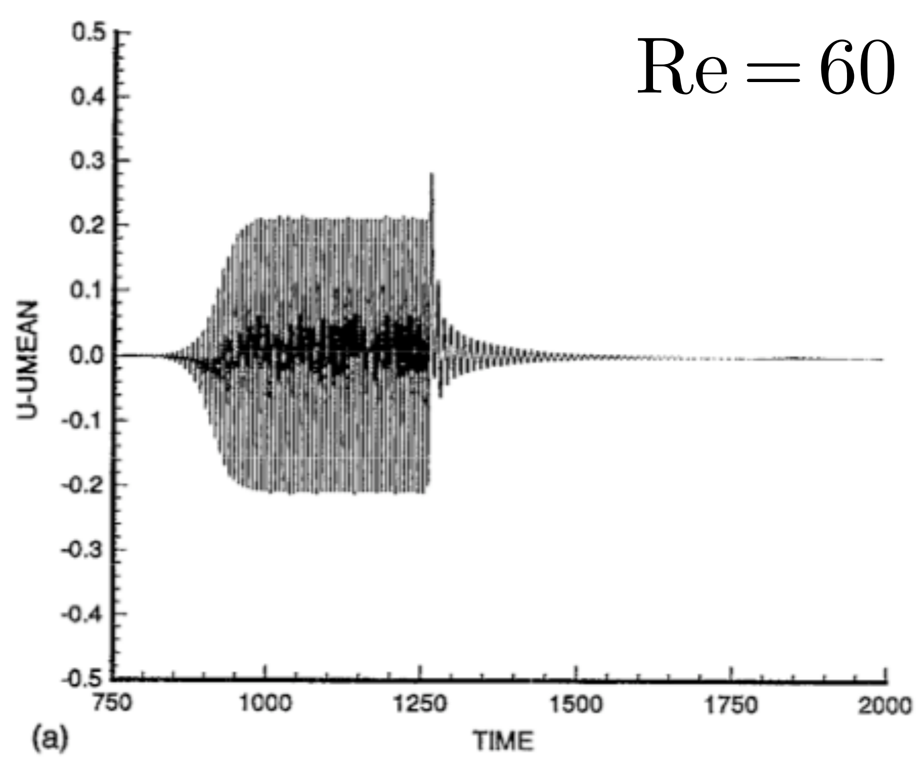


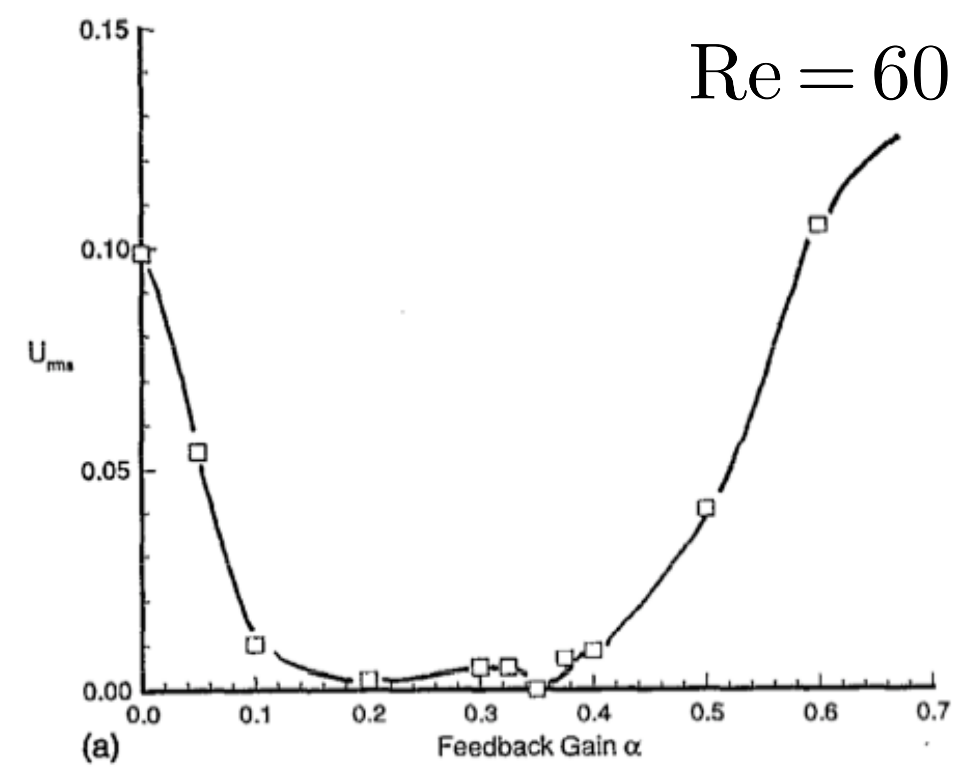
Previous closed-loop studies



Park et al., Phys. Fluids 1994:

- perform DNS and use a proportional feedback gain: $f(\theta,t) = g(\theta)\Gamma(t)$
  $\Gamma(t) = k u_s(t)$





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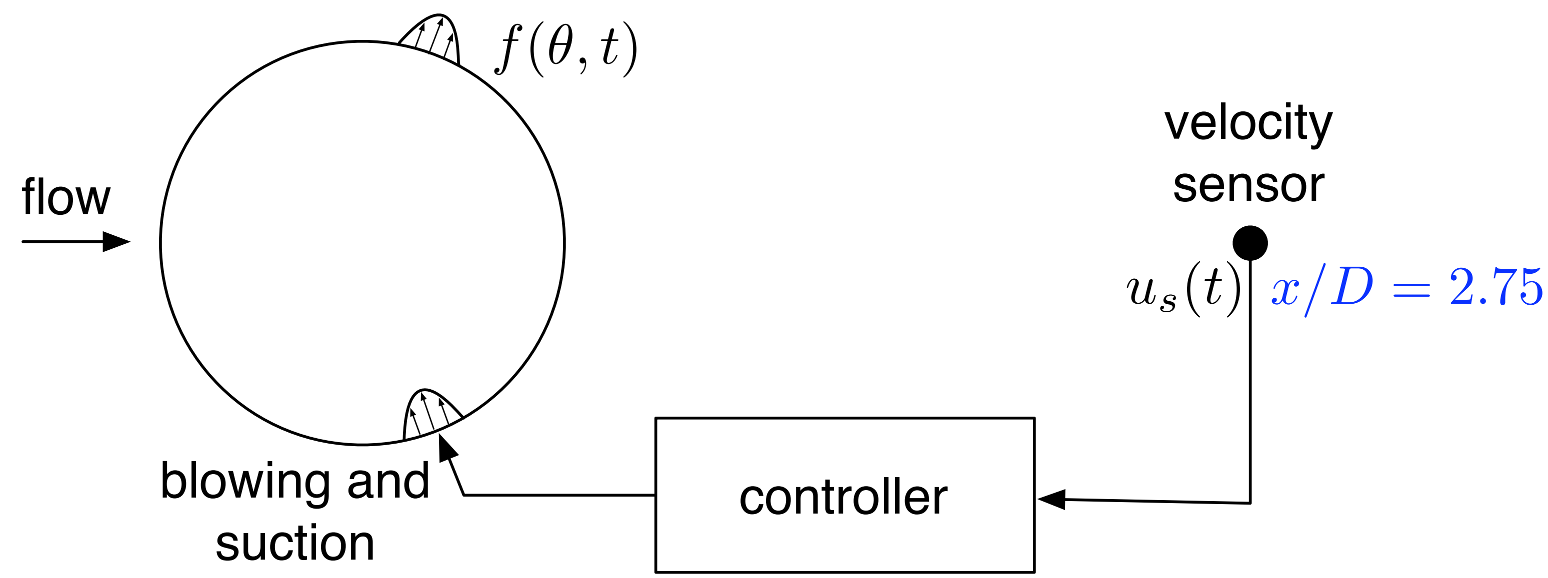
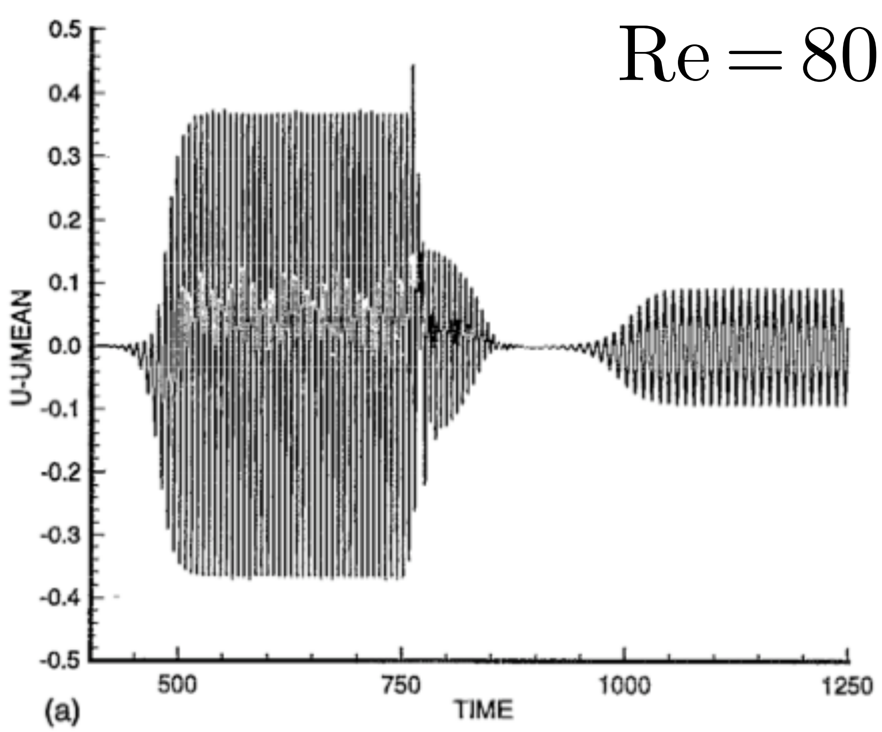


Park et al., Phys. Fluids 1994:

- used a proportional feedback gain in DNS: $$f(\theta, t) = g(\theta)\Gamma(t)$$ $$\Gamma(t) = k u_s(t)$$



- Roussopoulos (JFM 1993) found similar results in his experimental study
- he also used a proportional feedback gain
- and also observed a 'gain window', which shrinks with Reynolds number
- clearly there are limitations to using a proportional feedback gain

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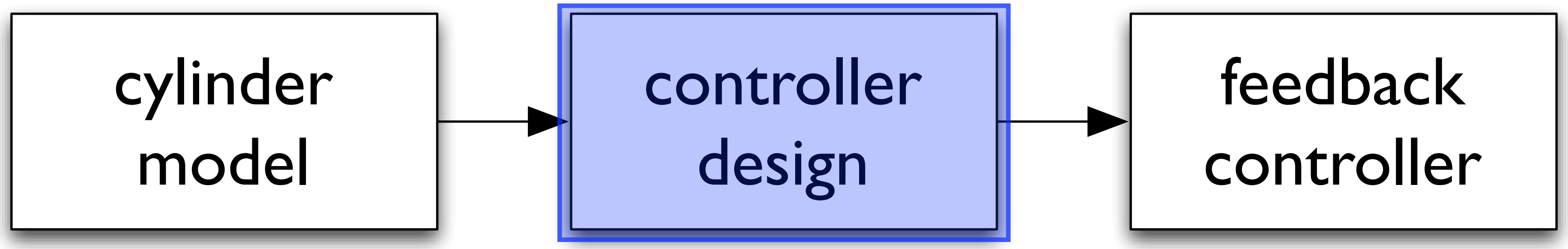
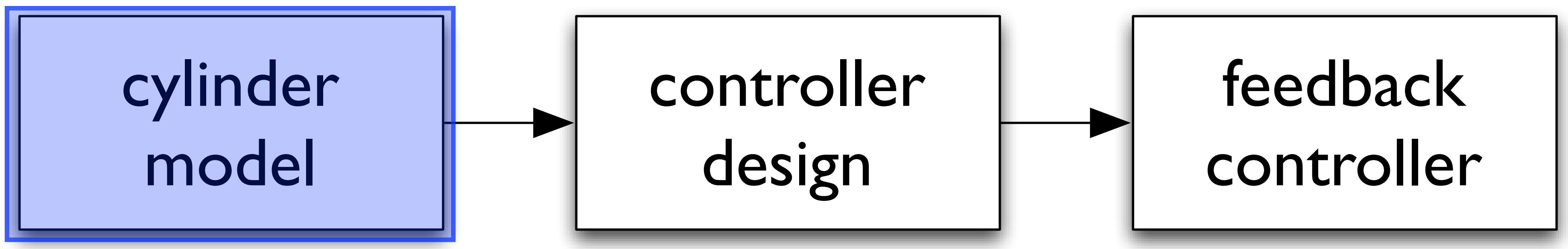
Current study

- these previous studies used simple feedback laws that were found using a combination of physical intuition and trial-and-error
- in the present work we propose a model-based control approach



- there are many different ways to design a model-based feedback controller
- the important feature of them all - distinguishing them from a proportional feedback gain - is that they are *dynamic* - just like the cylinder wake itself
- controller designed using $\mathcal{H}_\infty$ loop-shaping methods

Finding a reduced-order model



$$u(t) \rightarrow \begin{array}{|rcl|} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y(t) & = & Cx(t) \end{array} \rightarrow y(t)$$

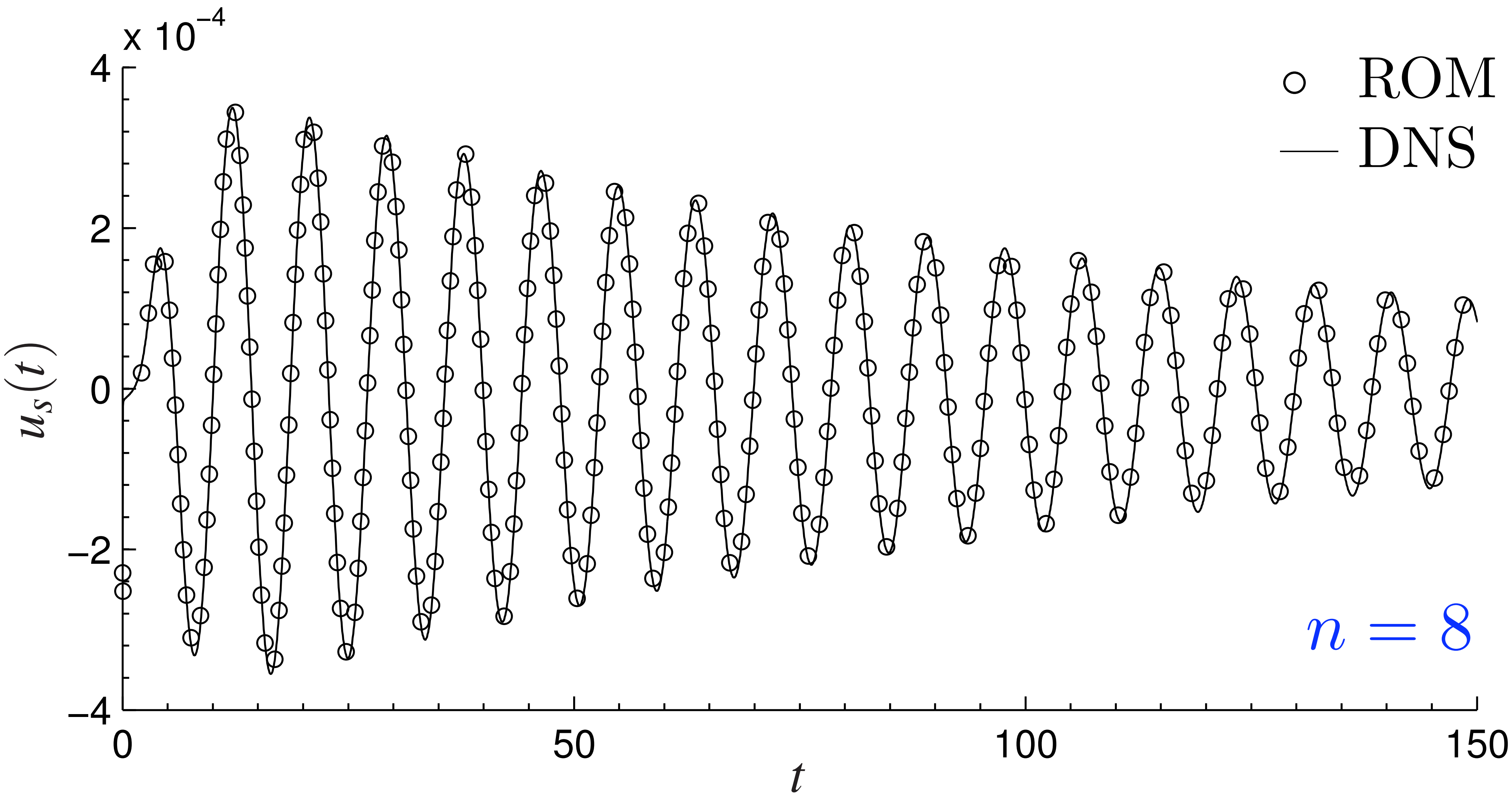
$u(t)$ is the *input*
$y(t)$ is the *output*
$x(t)$ is the *state*, $x(t) \in \mathbb{R}^n$

$$x = [x_1 \; x_2 \; \ldots \; x_n]^T, \; n \simeq 10$$

- state-space model found using the Eigensystem Realization Algorithm
- finding the *linear* dynamics of an unstable system is difficult - the growing amplitudes will give rise to non-linear, limit-cycling behaviour
- therefore we first find a reduced order model for a nearby, stable Reynolds number of Re=45

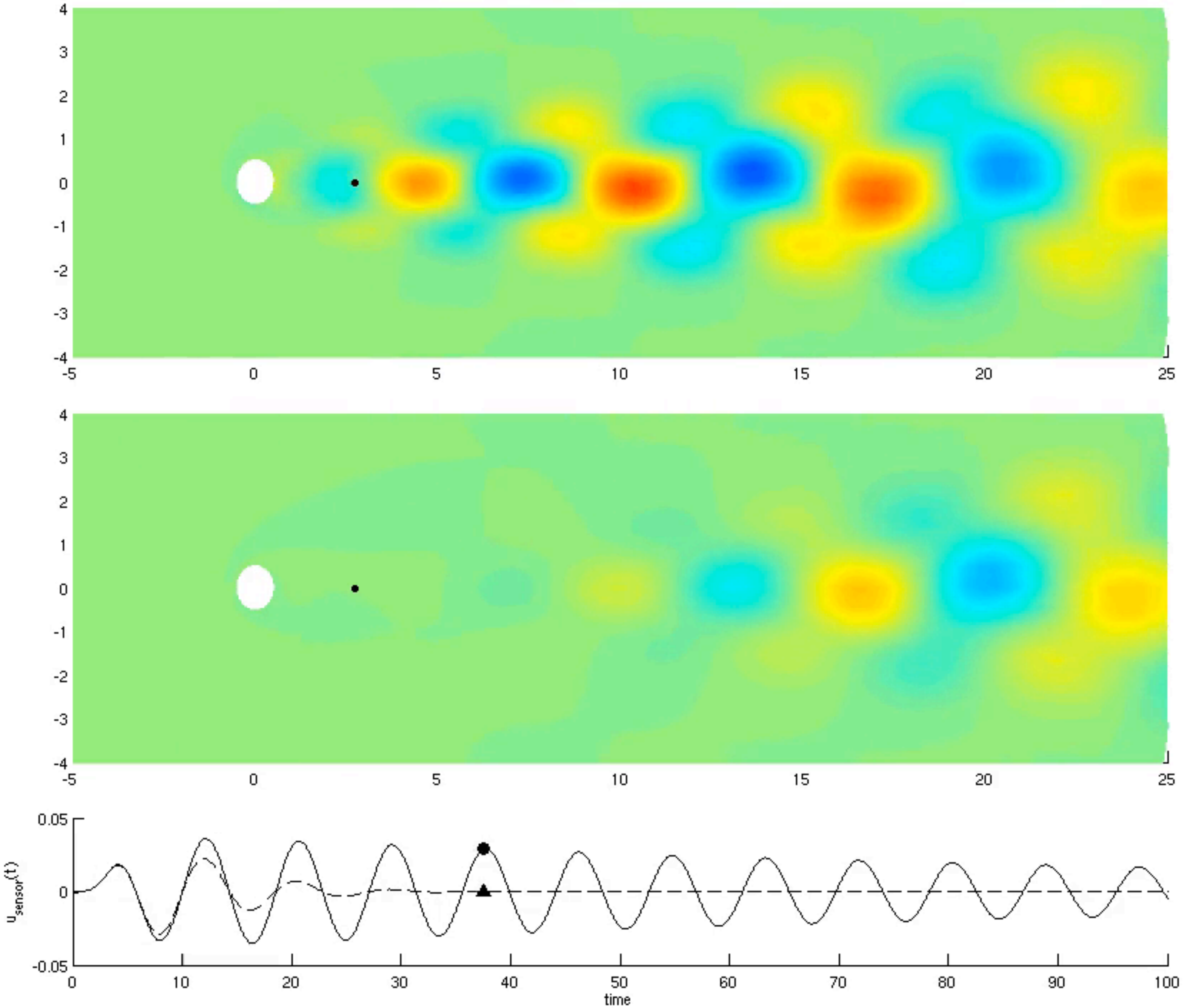
Reduced order model: Re=45

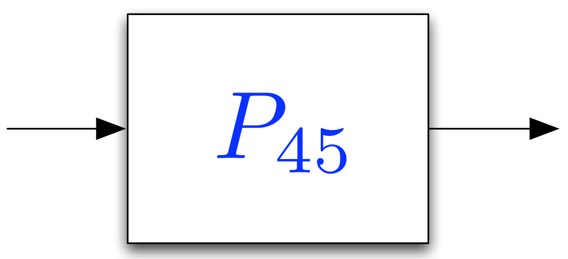
- here we validate the reduced-order model
- by comparing its impulse response to that found directly in DNS

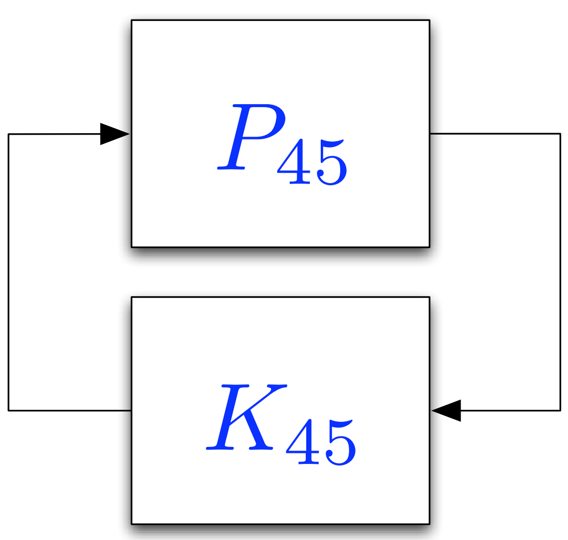


- a controller is first designed for Re=45
- but since the wake is stable, what does the controller actually do in this case?

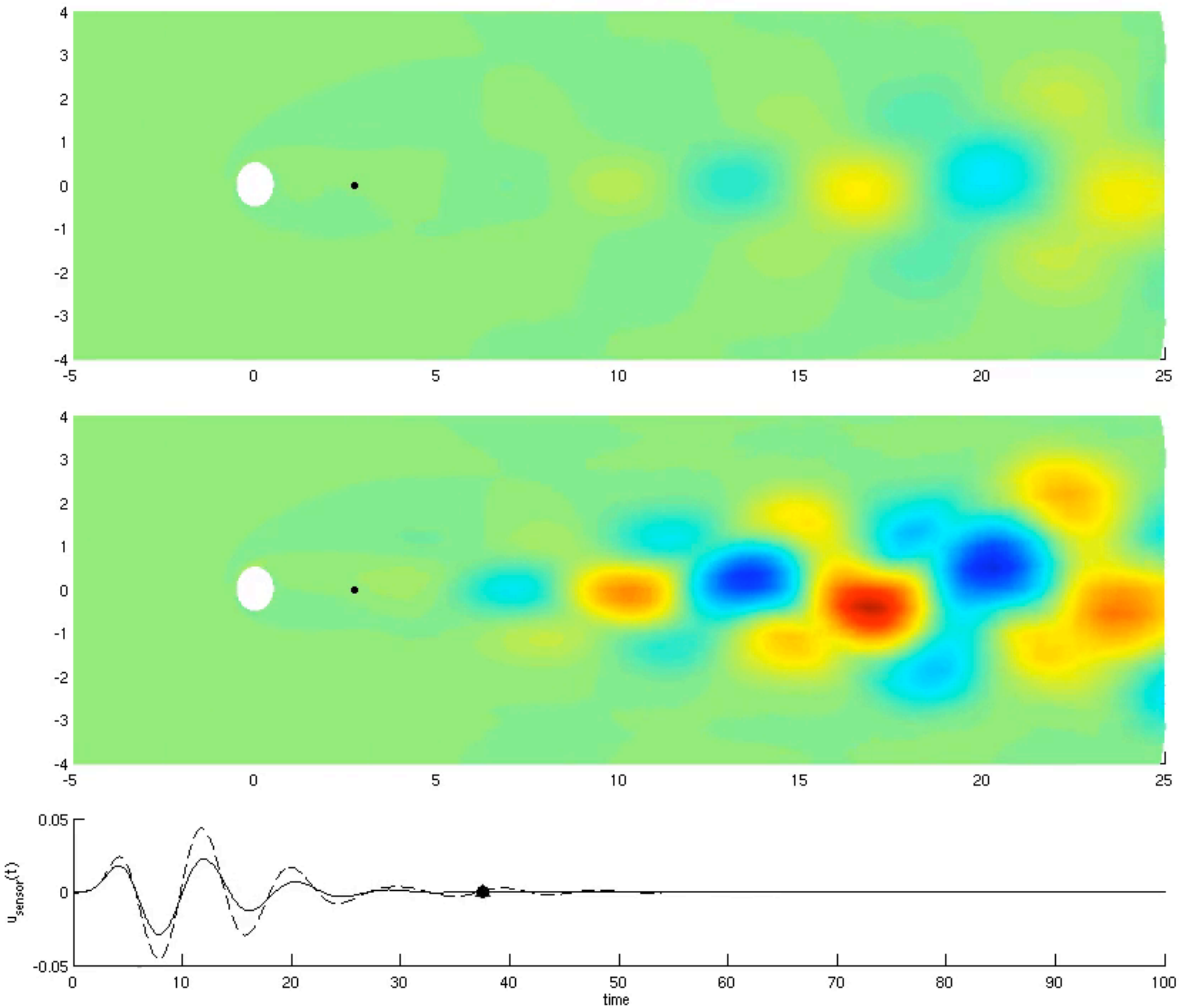
Closed-loop impulse response

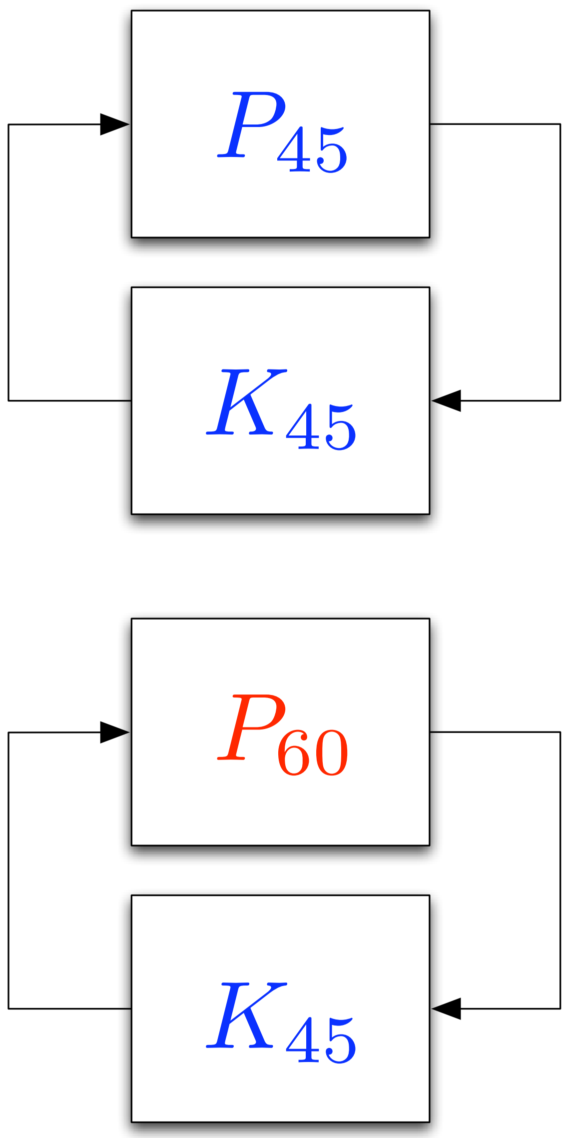






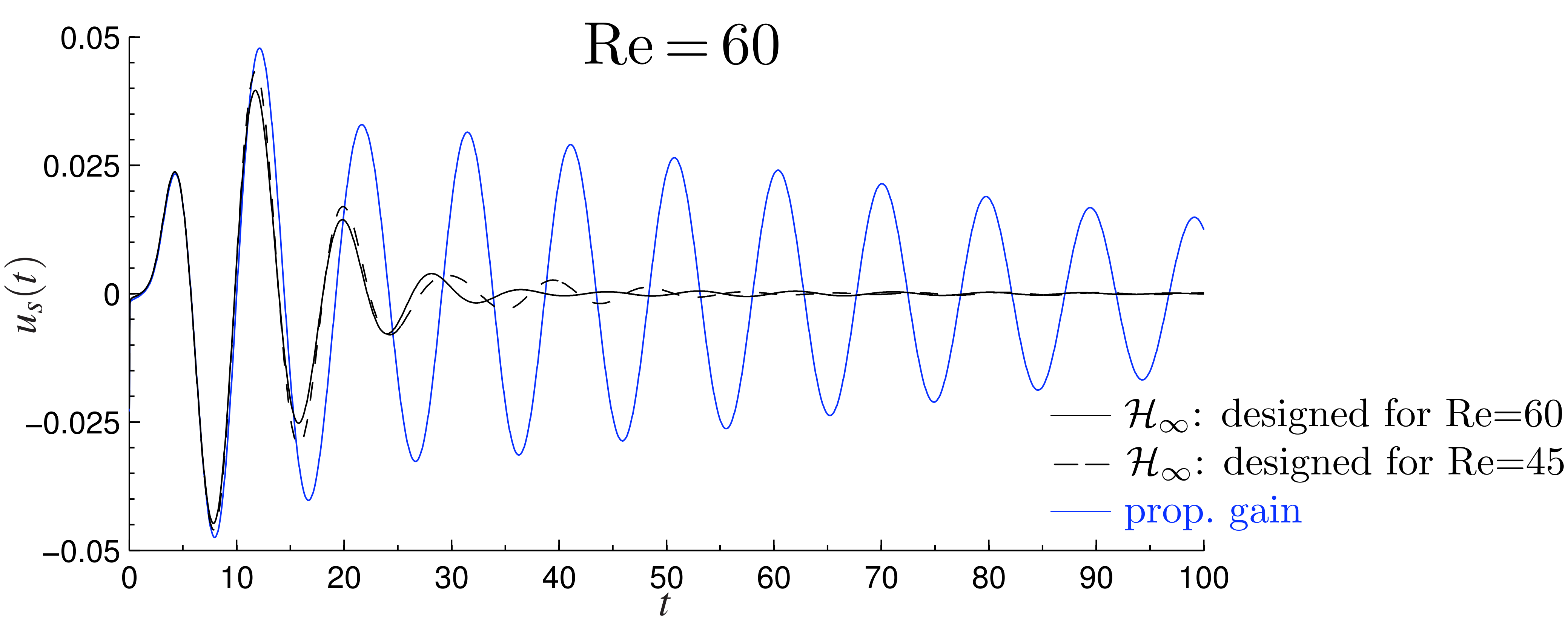
Closed-loop impulse response





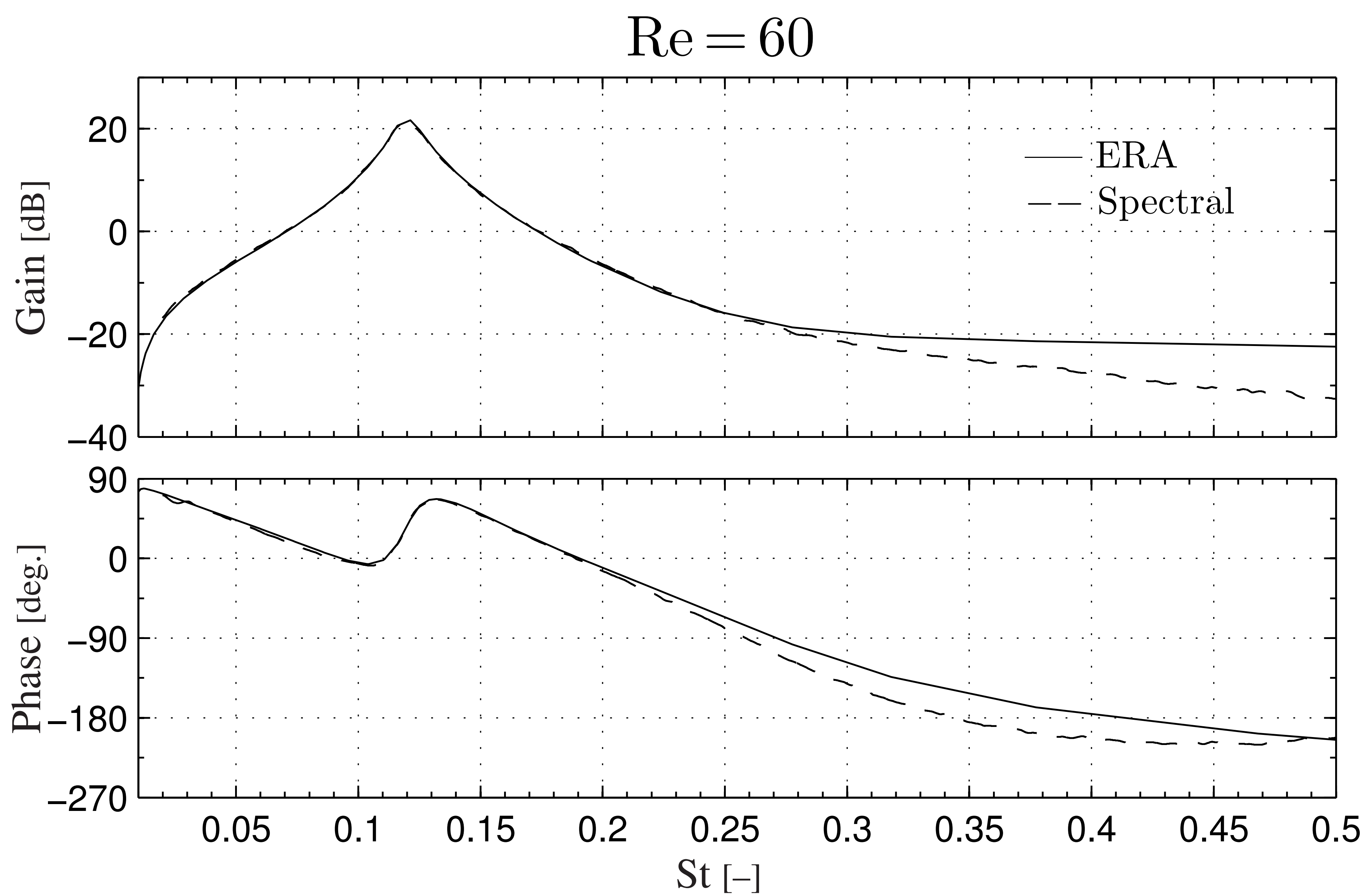
Closed-loop impulse response

- how does the model-based controller compare to a proportional feedback gain like that used by Park et al.?
- oscillations die away much more quickly - even when using the controller designed at Re=45

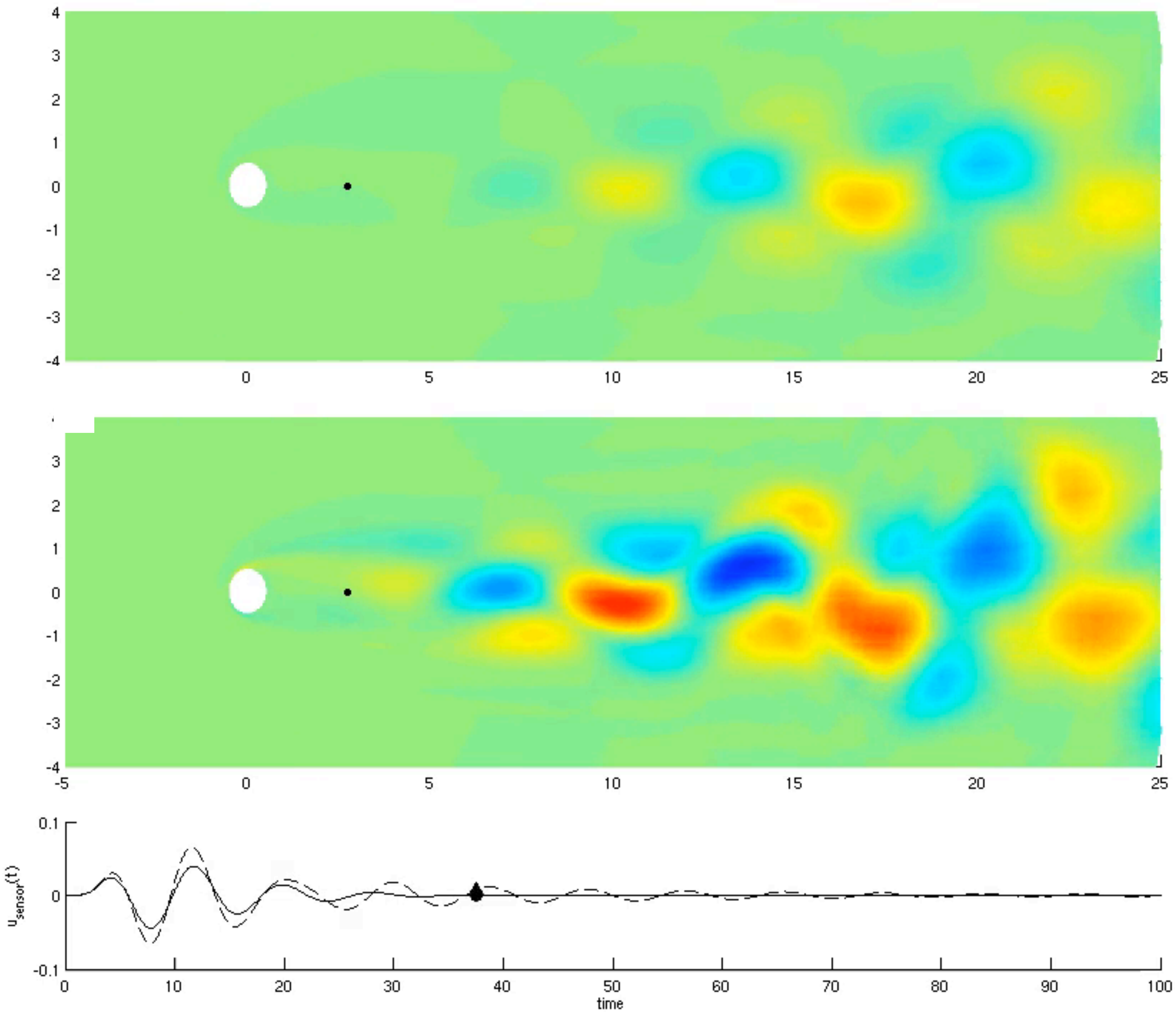


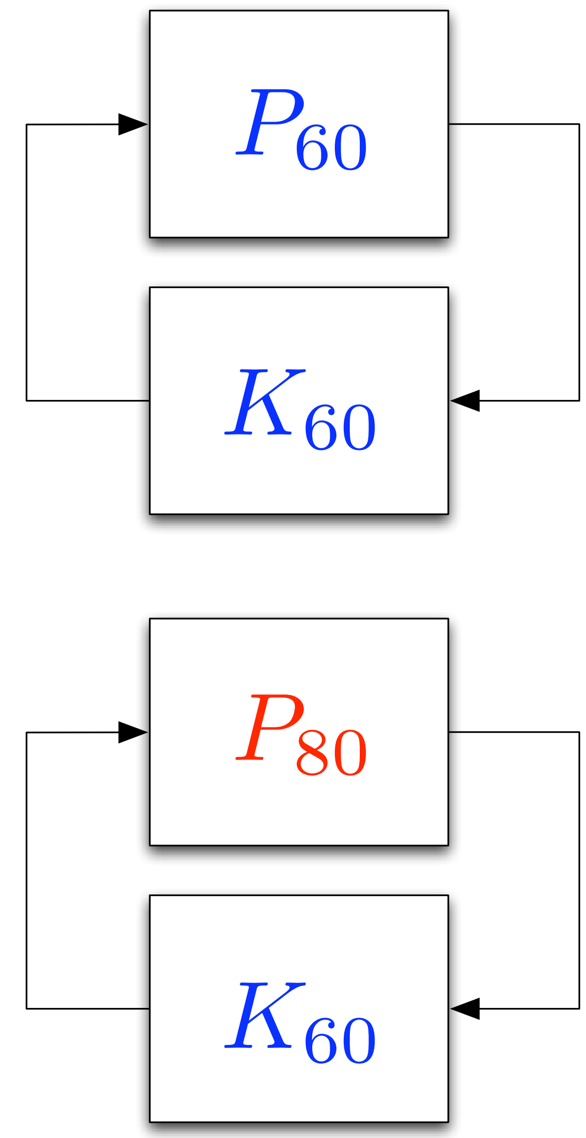
Frequency response

- with the Re=60 wake now stabilized, we can find a linear model for it
- the frequency response (or 'transfer function') is a very concise way of validating this model
- it is also very useful for controller design



Closed-loop impulse response





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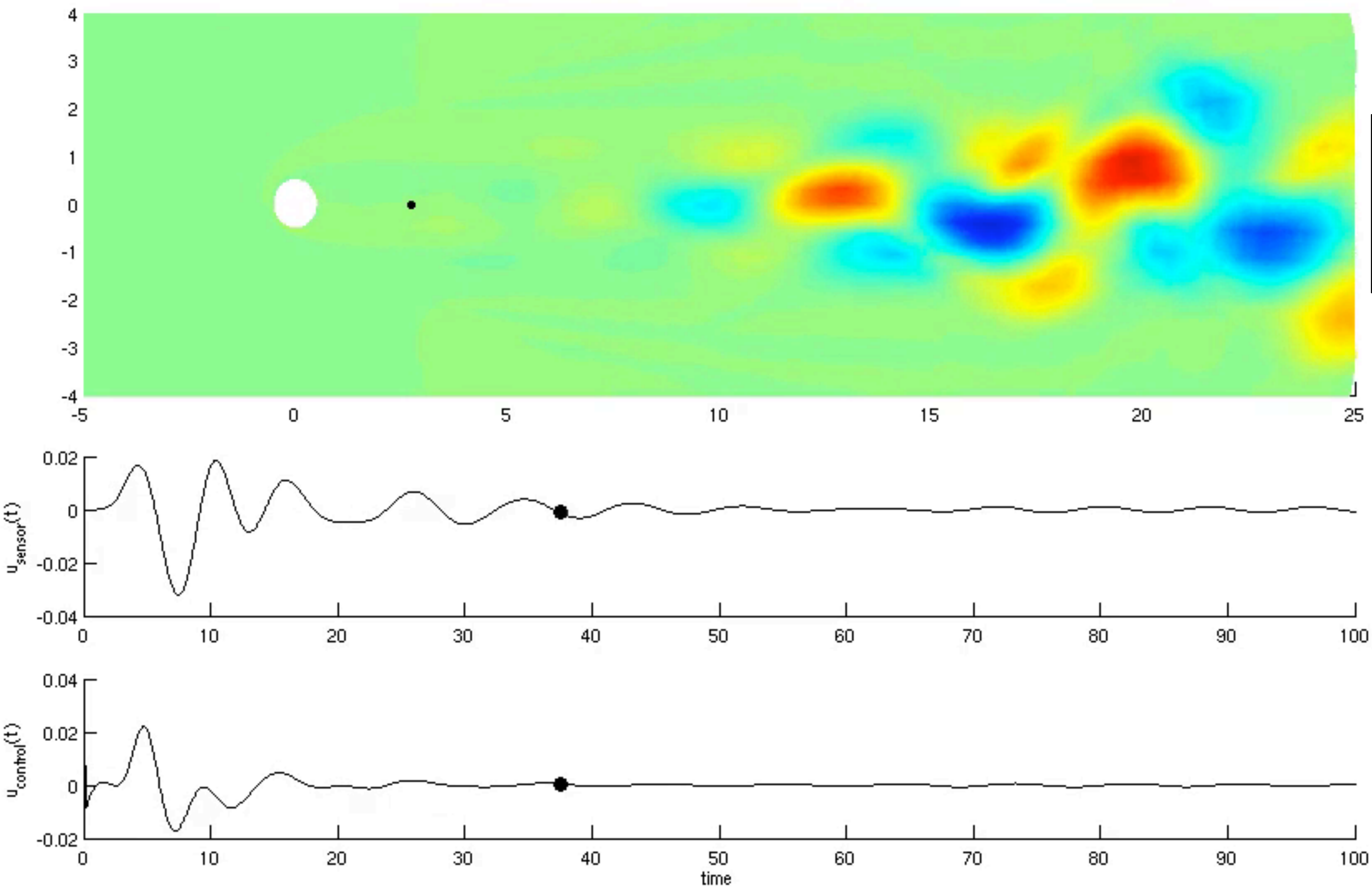
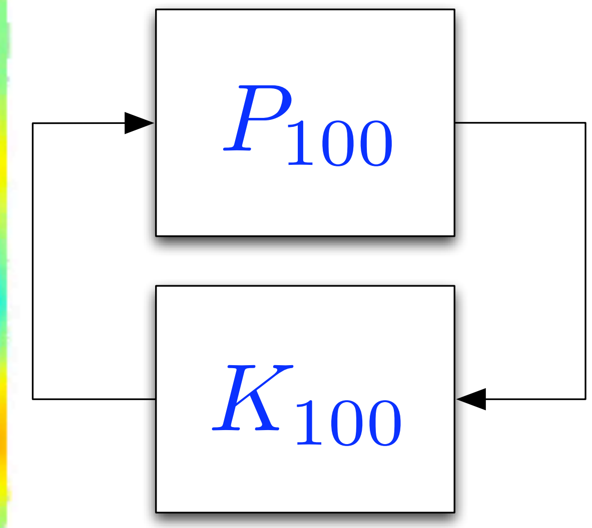
Control at higher Reynolds numbers

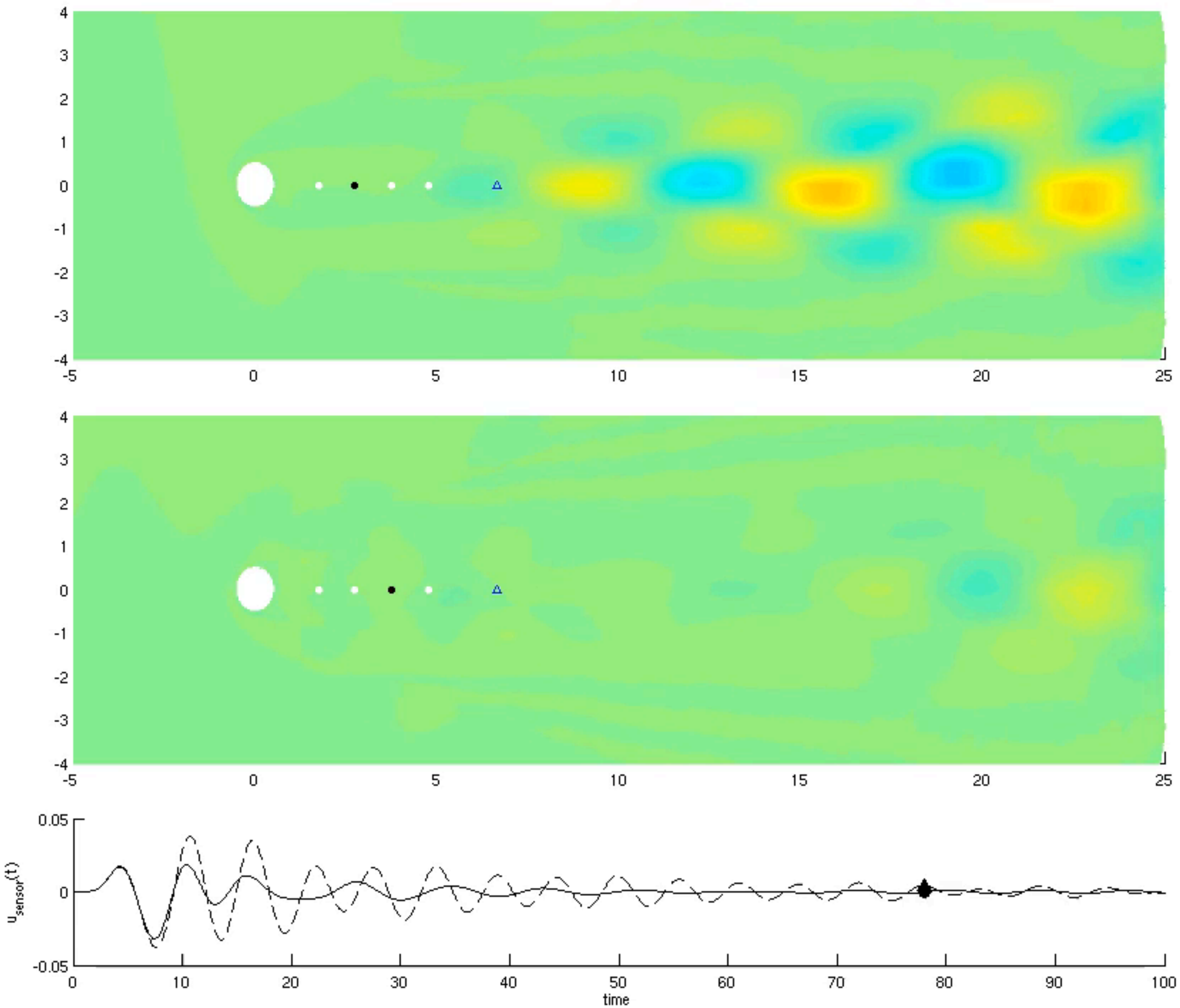
- by Re=100, control with a single sensor becomes more difficult
- this is caused by a larger region of absolute instability

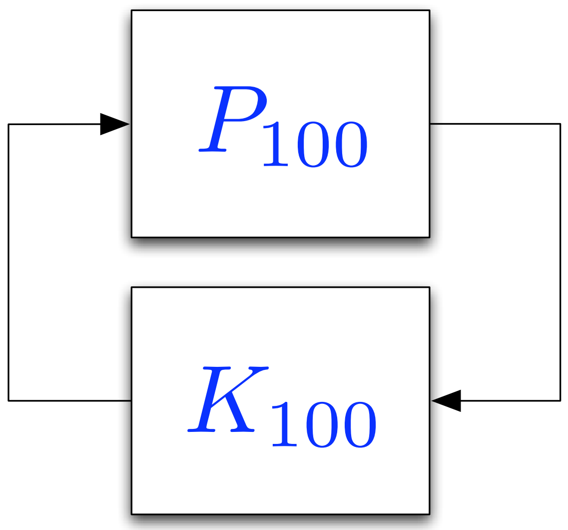




Control at higher Reynolds numbers

- so what if we put the sensor further downstream?





Two conflicting requirements

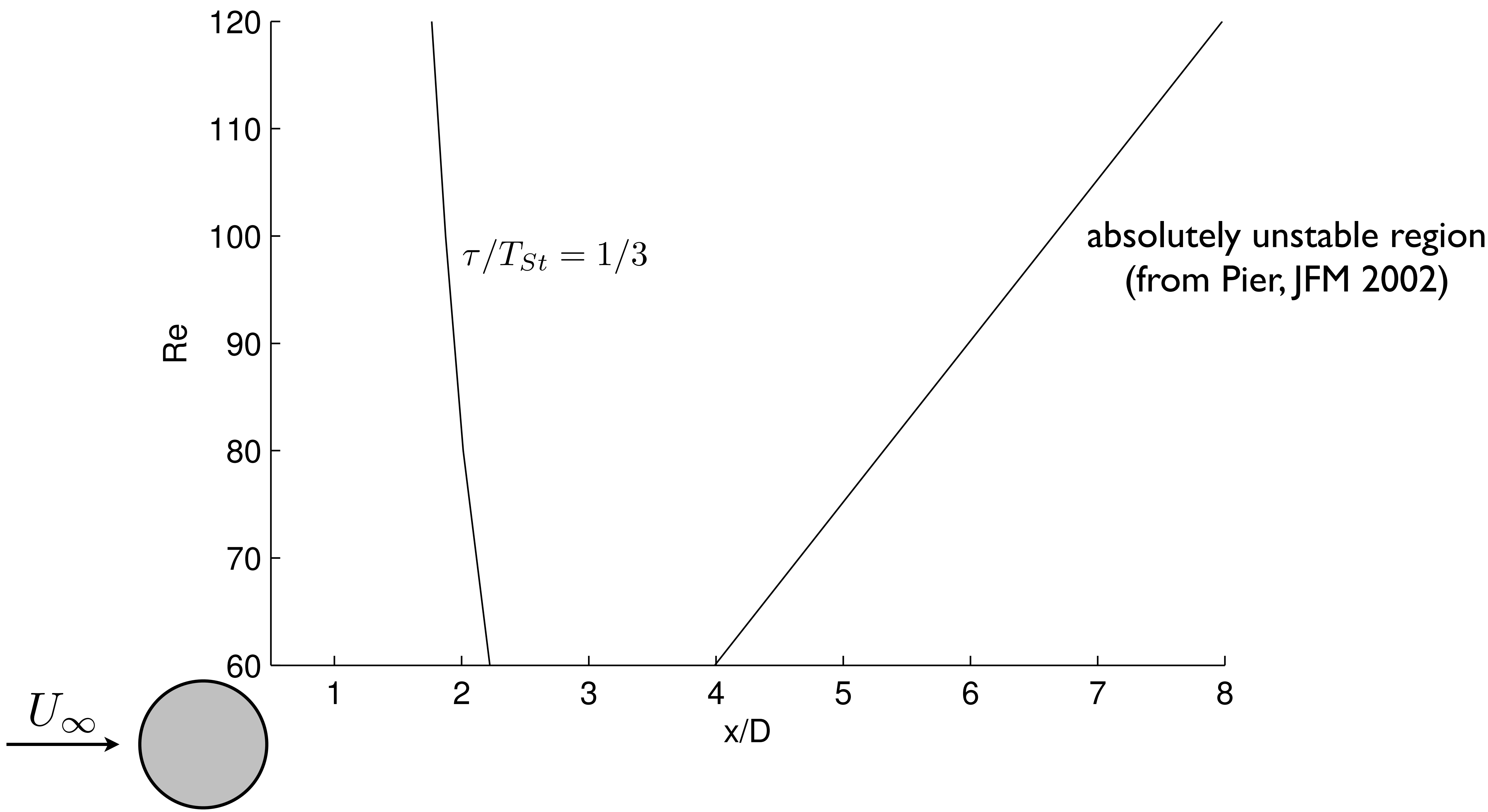
- for the sensor location, we want two things:

1. reasonably 'new' information
    - i.e. a small time delay
    - means putting the sensor near the cylinder surface

2. information from the unstable part of the wake
    - means putting the sensor further downstream as Re increases

- these two requirements are conflicting - improving 1. makes 2. worse, and vice-versa

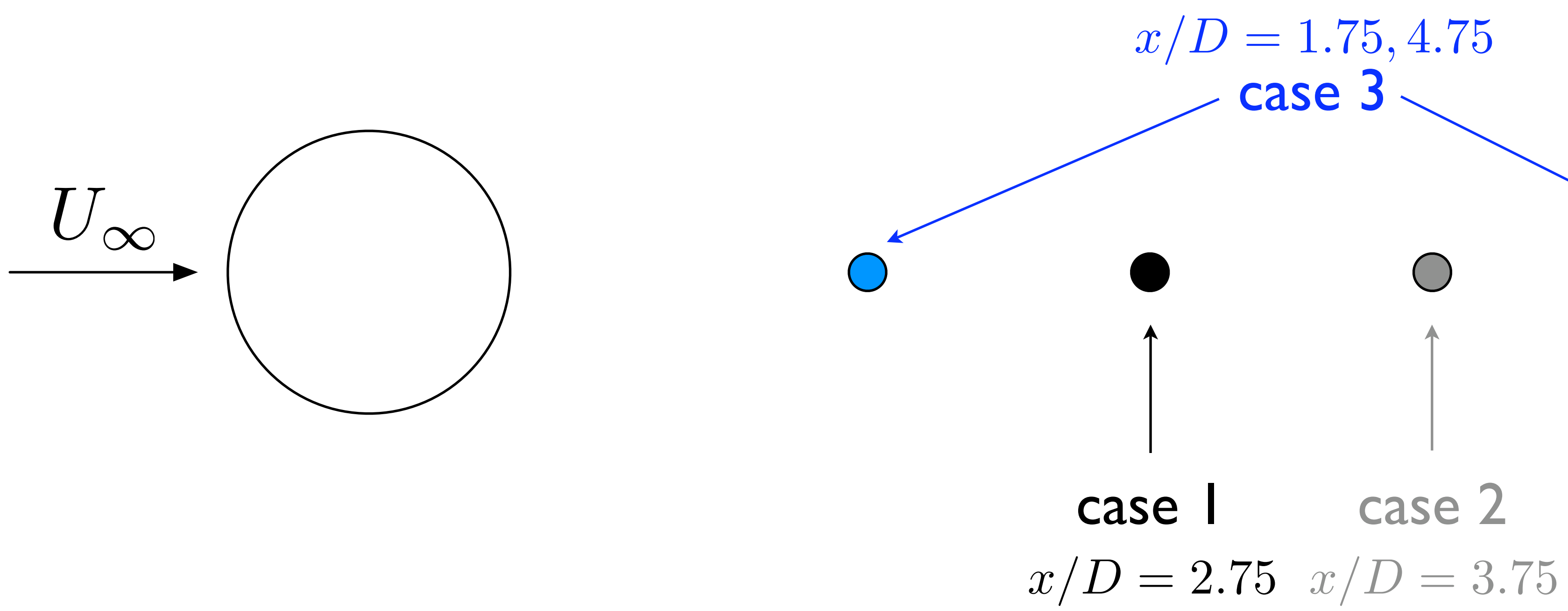
Control at higher Re



- these two locations grow further apart as the Reynolds number increases
- and so meeting both requirements with a single sensor is increasingly difficult

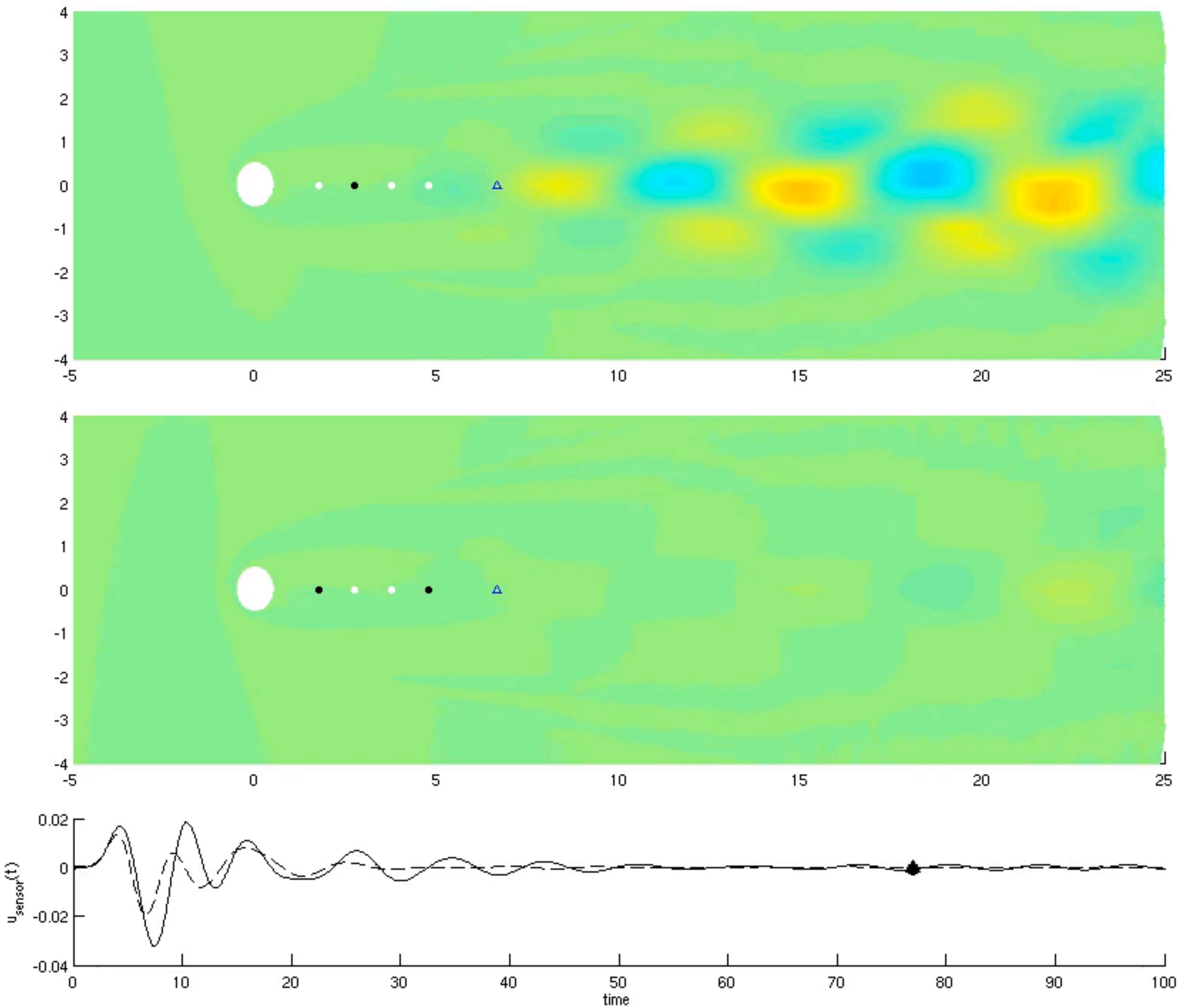
Two sensor case

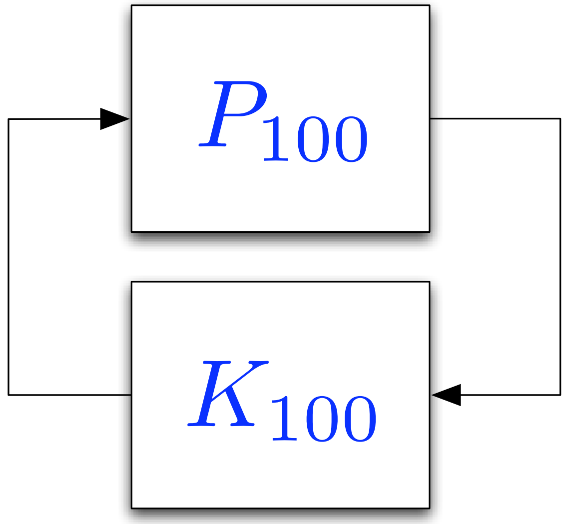
- so can we improve the performance by using two sensors?



Two sensor case

- comparison: single sensor vs two sensors





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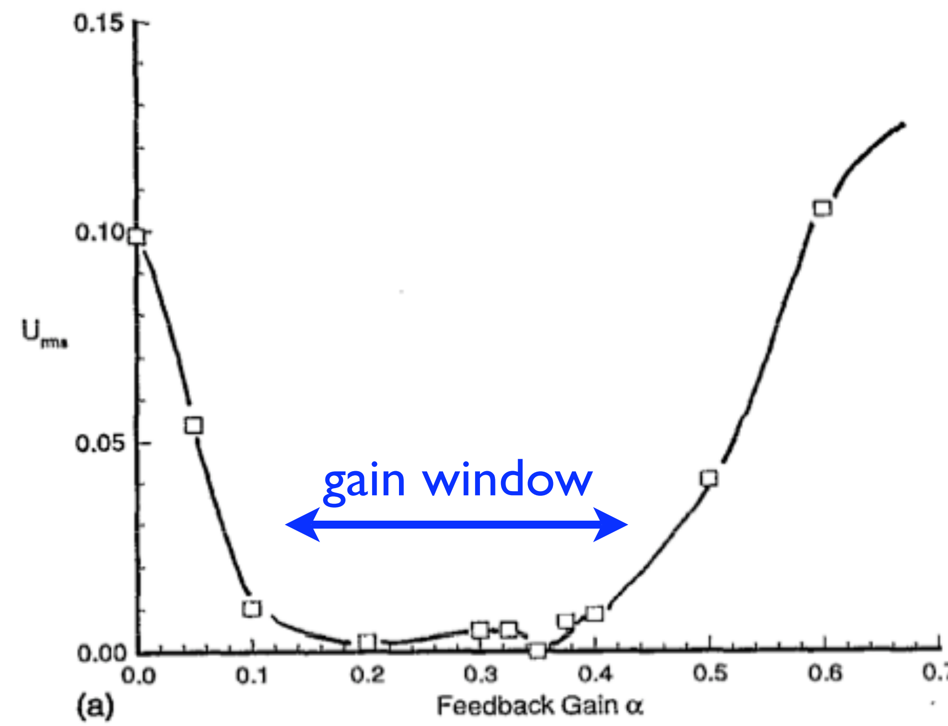
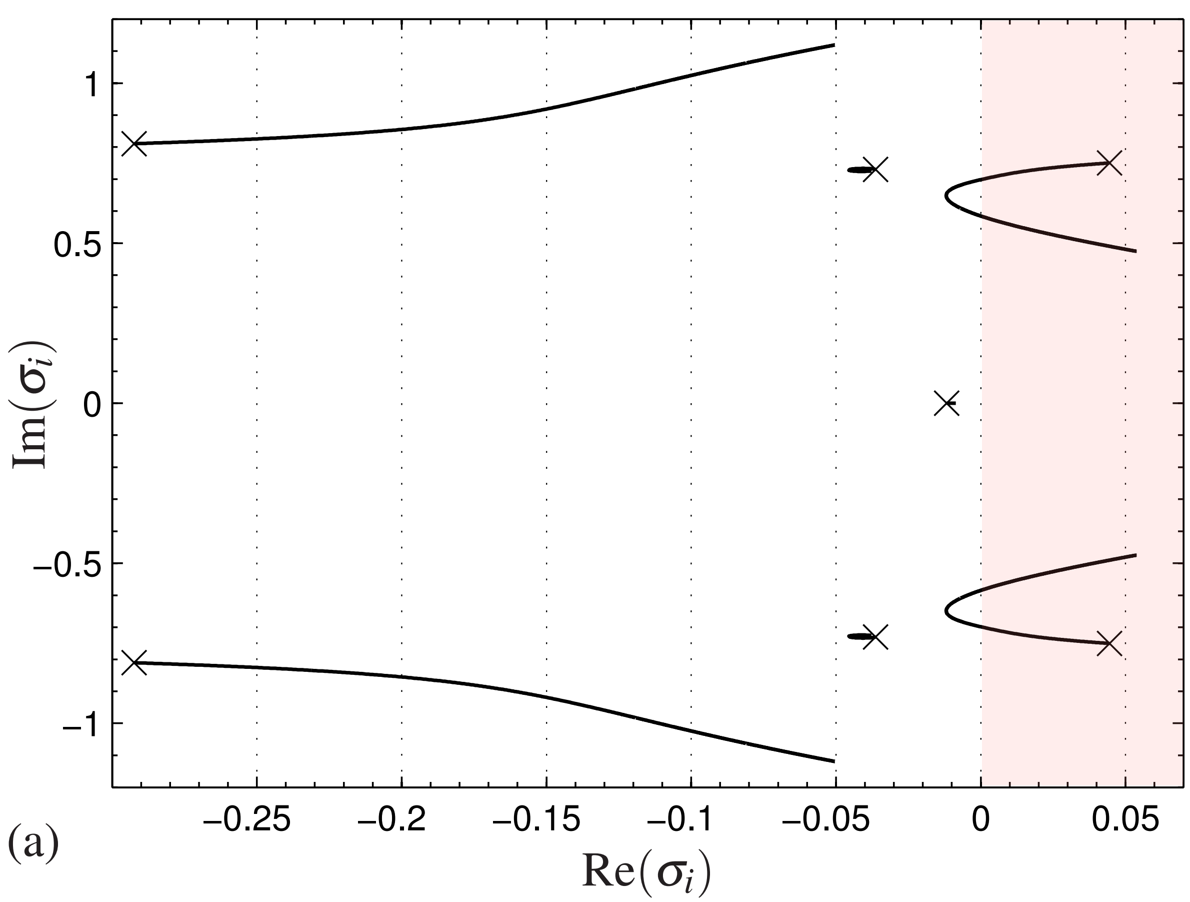
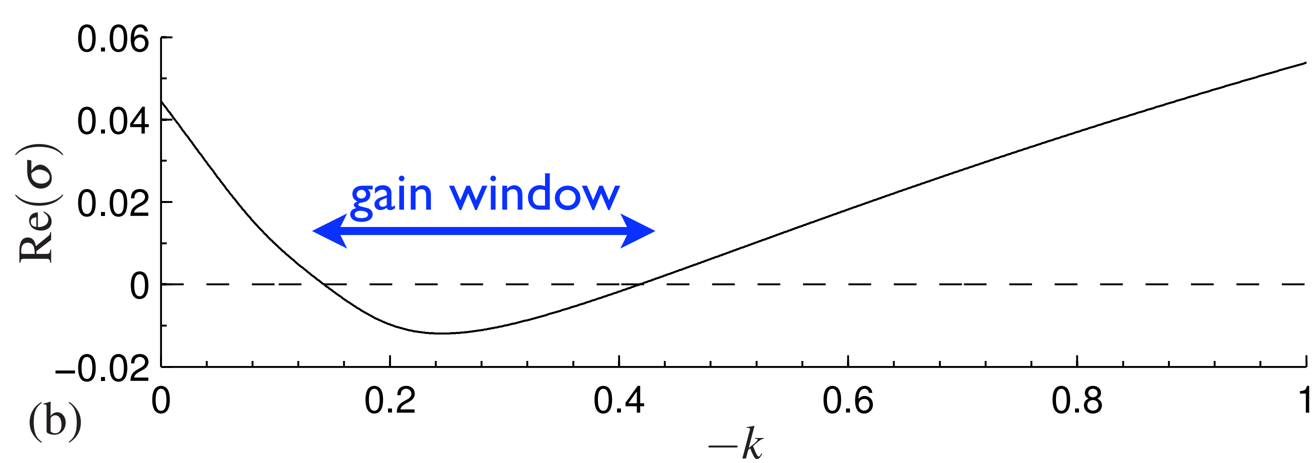
Park et al., Phys. Fluids 1994:

$\mathrm{Re} = 60$



Reduced-order model: root locus





Conclusions

- a model-based controller performs much better than a trial-and-error based proportional feedback gain
- at higher Reynolds numbers, control with a single sensor becomes more and more difficult. This is caused by two conflicting requirements for the sensor location:
  1. near the cylinder to ensure a small time delay
  2. far enough downstream to have enough information about the unstable part of the wake

Acknowledgement

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