UNIVERSITY OF CAMBRIDGE

Forced flame dynamics and self-excited instabilities of laminar premixed flames

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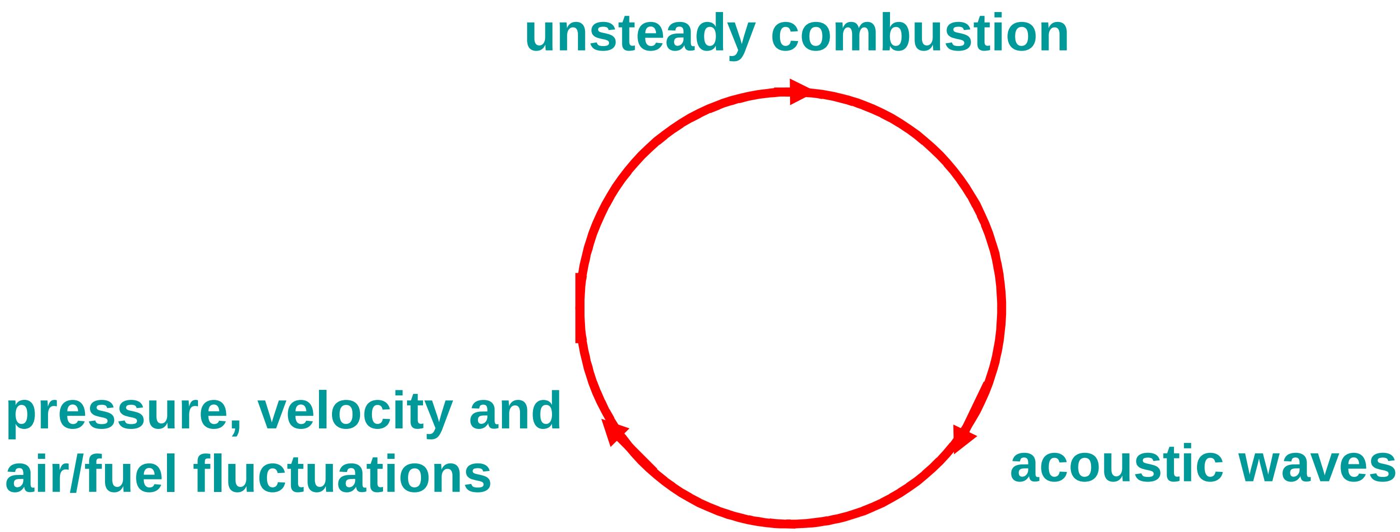
CAMBRIDGE COMBUSTION RESEARCH CENTRE

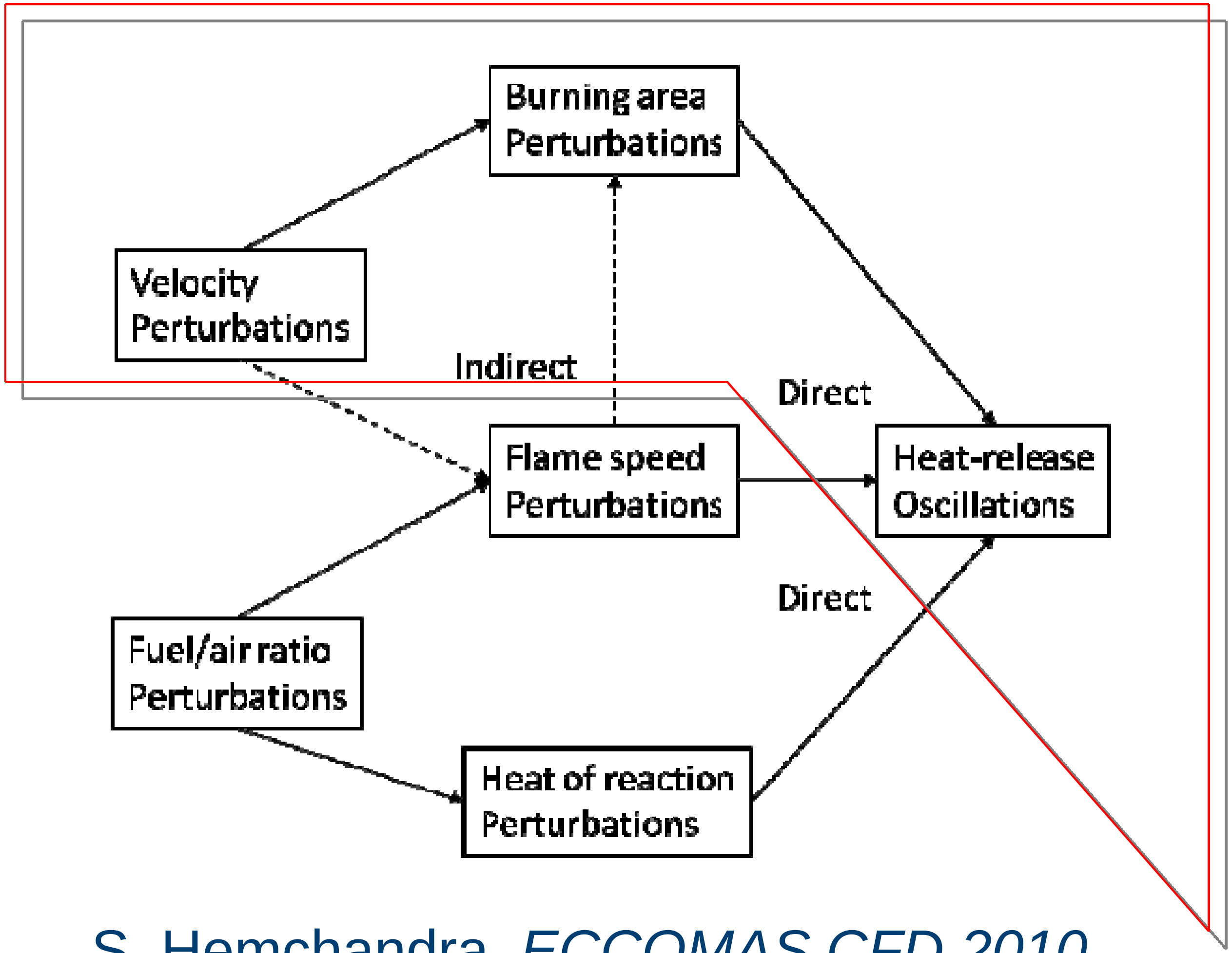
Thermoacoustic instabilities

Self-excited oscillations due to the coupling shown below



Unsteady heat release: mechanisms

Premixed flames
have unsteady q’
due to u’ only.

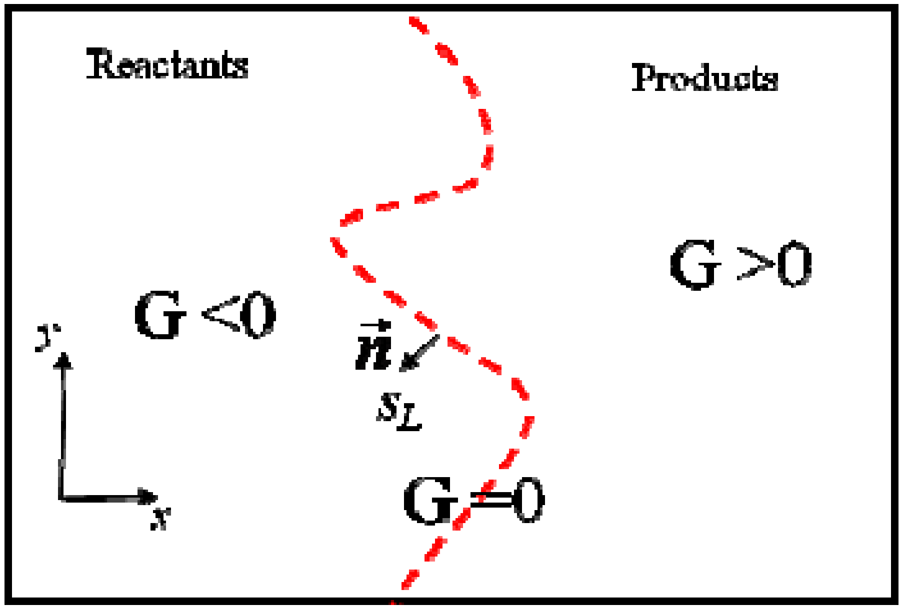


S. Hemchandra, *ECCOMAS CFD 2010*

Premixed flame model: level set approach

The flame is modelled as a thin moving boundary between reactants and products

Flame represented as a level set of a function G(x,y,t).

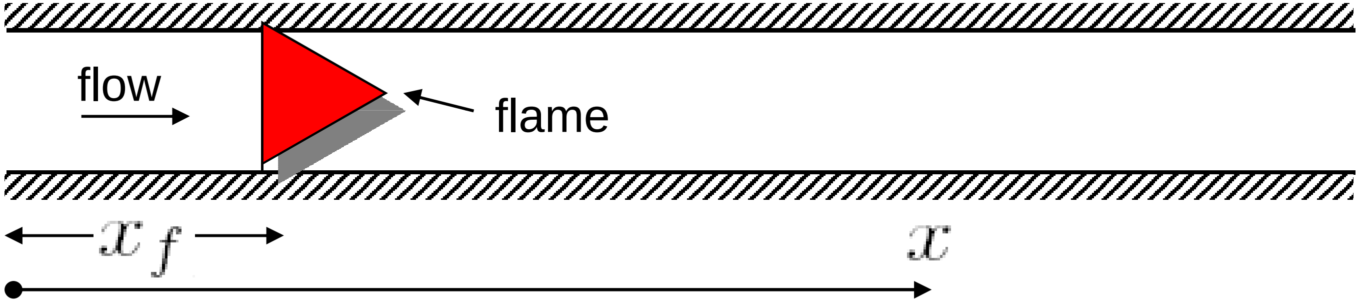


$$\frac{\partial G}{\partial t} + \bar{u}_{flame} \bullet \nabla G = 0$$

$$\frac{\partial G}{\partial t} + \left(\bar{u}(\bar{x},t) - s_L \frac{\nabla G}{|\nabla G|} \right) . \nabla G = 0$$

Linear acoustics

Diagram of premixed flame in a tube



Non-dimensional governing equations

$$F_1 \equiv \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0,$$

$$F_2 \equiv \frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} + \zeta p - \beta \delta_D (x - x_f) = 0$$

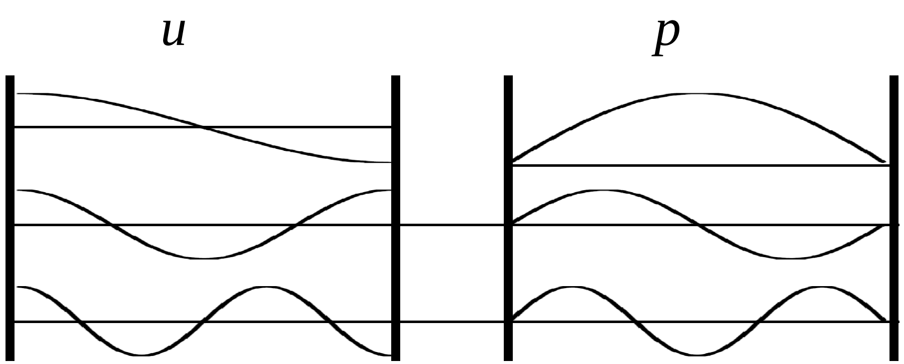
where

$$\beta = \frac{(\gamma - 1)\tilde{\dot{Q}}_{\tilde{x}f}}{\gamma M p_0 c_0}$$

Discretization into basis functions

$$u(x,t) = \sum_{j=1}^{N} \eta_j(t) \cos(j\pi x),$$

$$p(x,t) = -\sum_{j=1}^{N} \left(\frac{\dot{\eta}_j(t)}{j\pi}\right) \sin(j\pi x),$$



Flow field: Simplified model of convective disturbances

Acoustic velocity disturbances at the burner lip are convected downstream at a characteristic velocity (Uc).

$$\varepsilon = \frac{u'}{u_0}$$

$$u(x,t) = 1 + \varepsilon \cos(kx - \omega t)$$

$$k = \frac{\omega}{u_c} = \frac{\omega}{u_0}\frac{u_0}{u_c} = K\left(\frac{\omega}{u_0}\right)$$

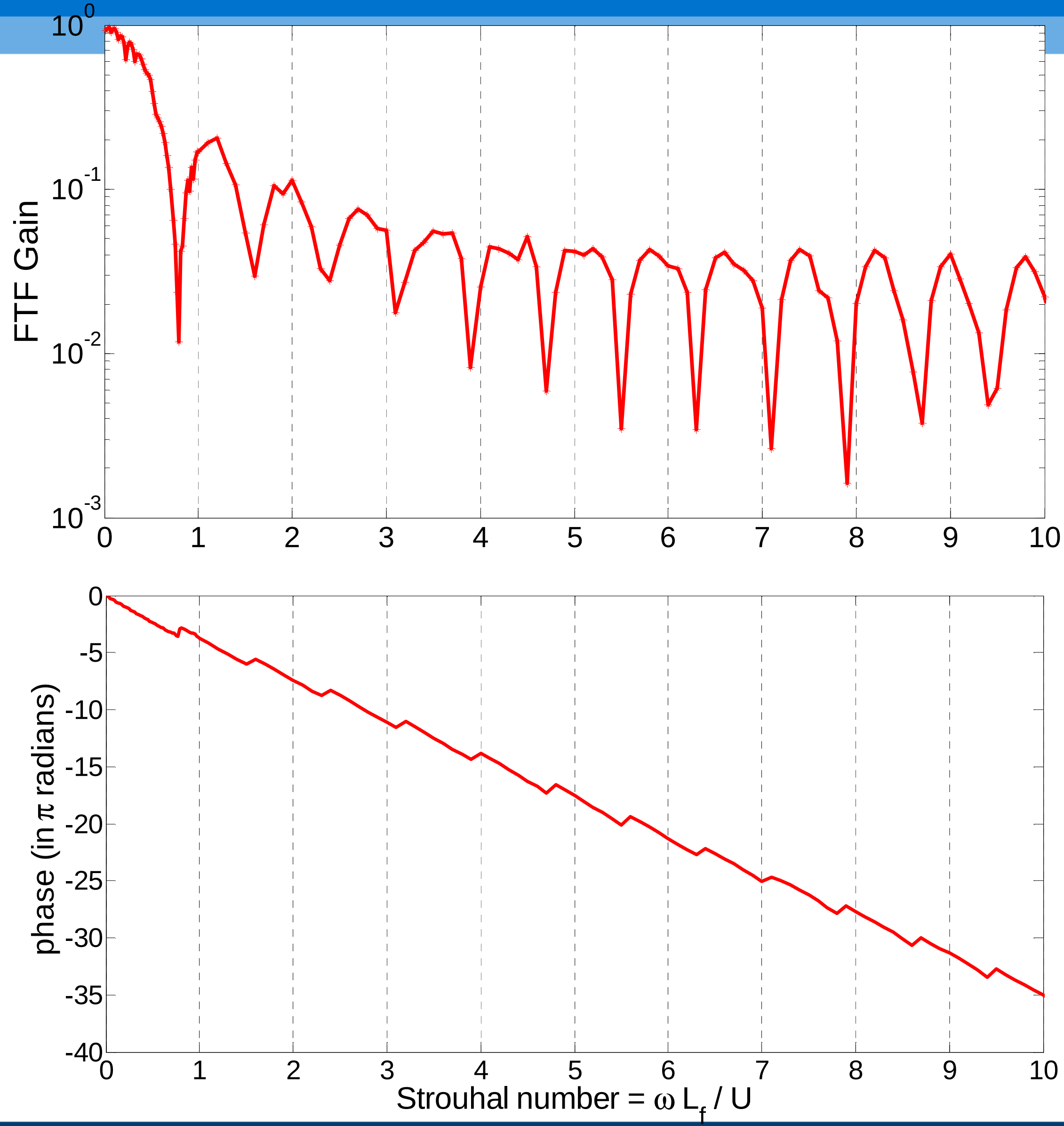
K is a parameter that can be varied in the model

S. Ducruix et. al., *Proc. Comb. Inst. 2000*

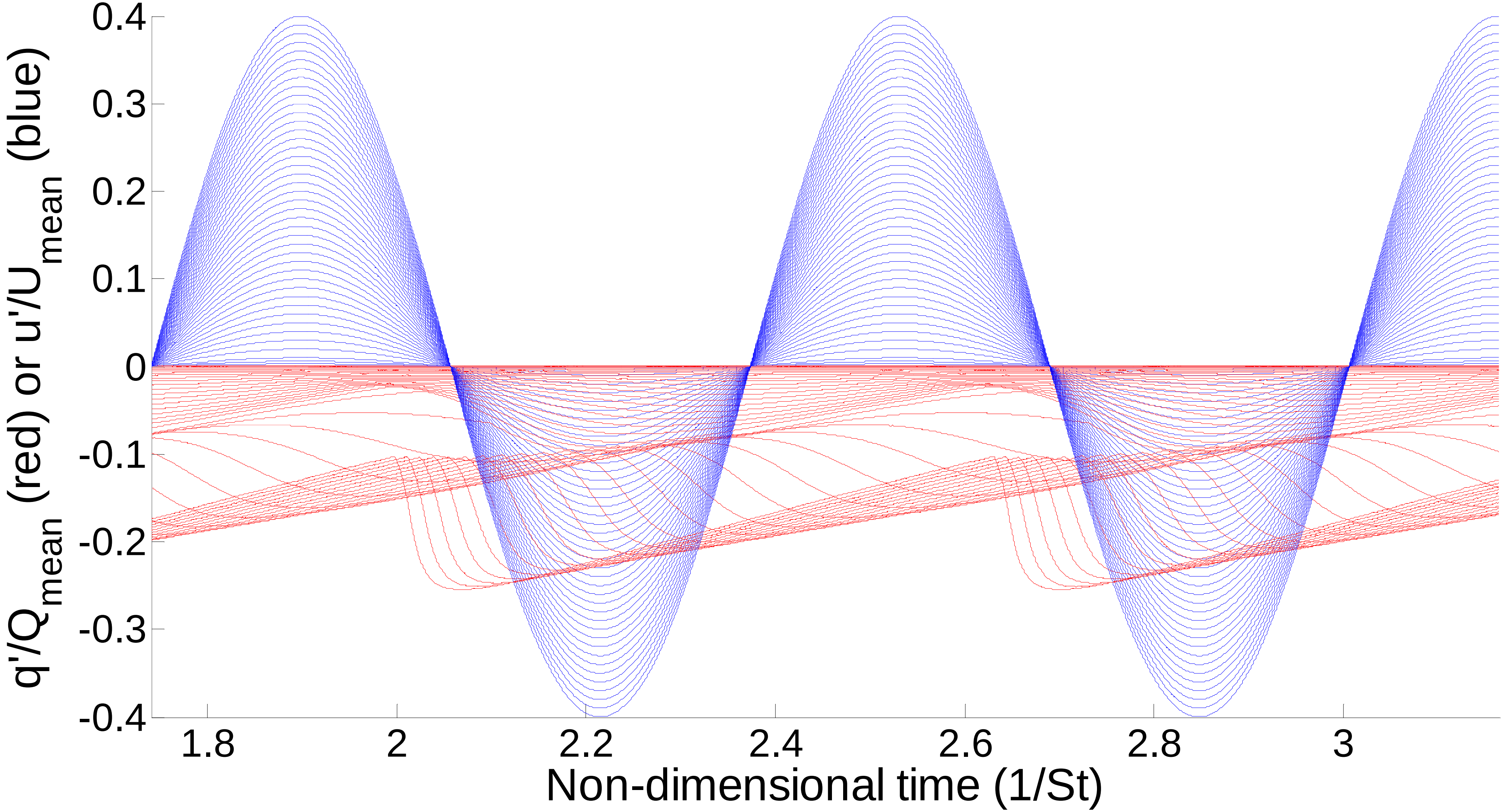
Linear response: Flame transfer function gain and phase

$$Gain = \mathrm{Re}\left(\frac{\frac{q'}{Q}}{\frac{u'}{U}}\right)$$

$$Phase = Angle\left(\frac{\frac{q'}{Q}}{\frac{u'}{U}}\right)$$



Nonlinear response of unsteady heat release at St = 1.6



Nonlinear stability criterion for periodic solutions

The stability of periodic solution can be examined as follows (M. P. Juniper)

$$u = r\cos(\omega t + \phi)$$

$$q = \sum_j a_j \cos(j\omega t + \phi_j)$$

$$\oint \frac{dE}{dt} dt = \pi r (a_1 \sin(\omega\tau) - r\zeta\omega)$$

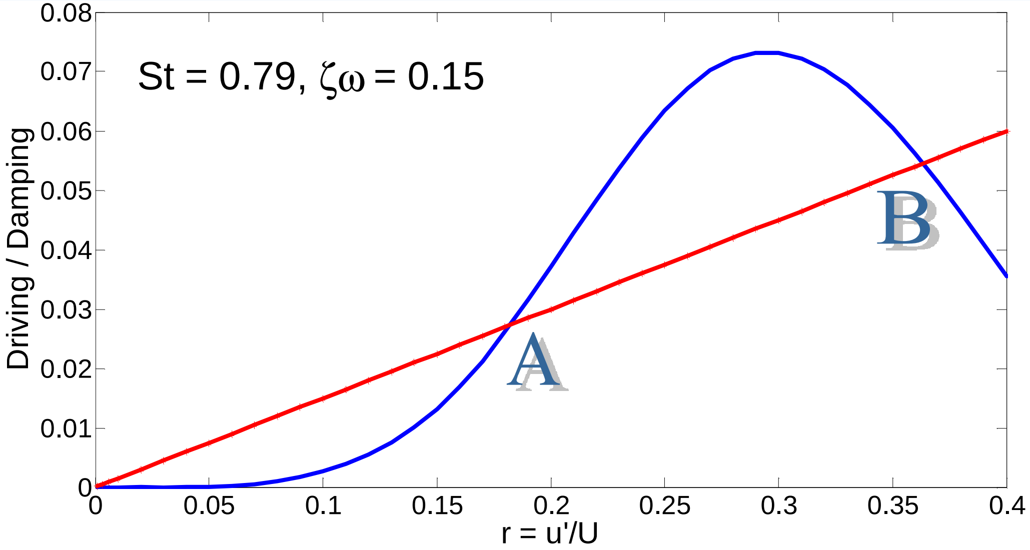
where

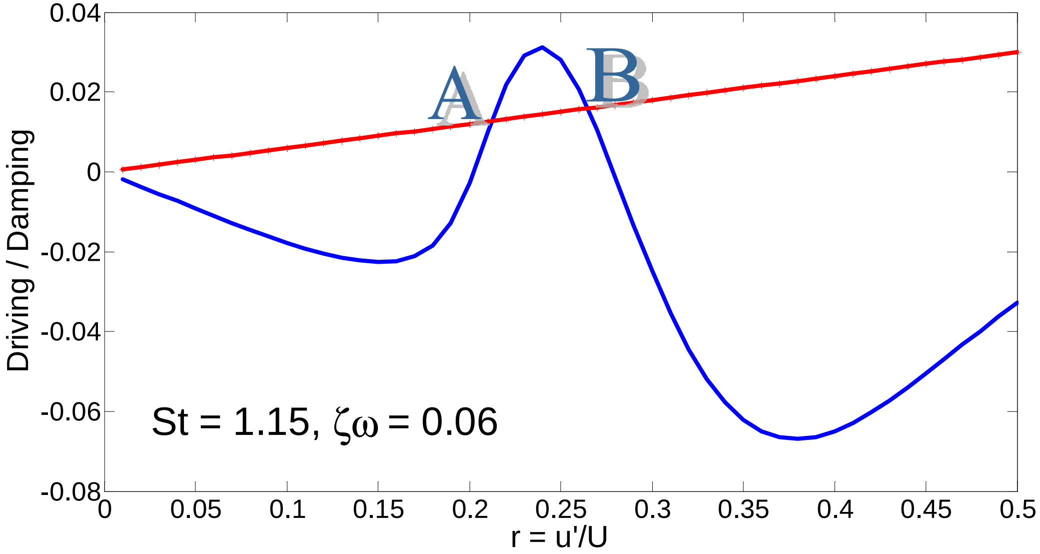
$$\tau = \frac{\phi - \phi_1}{\omega}$$

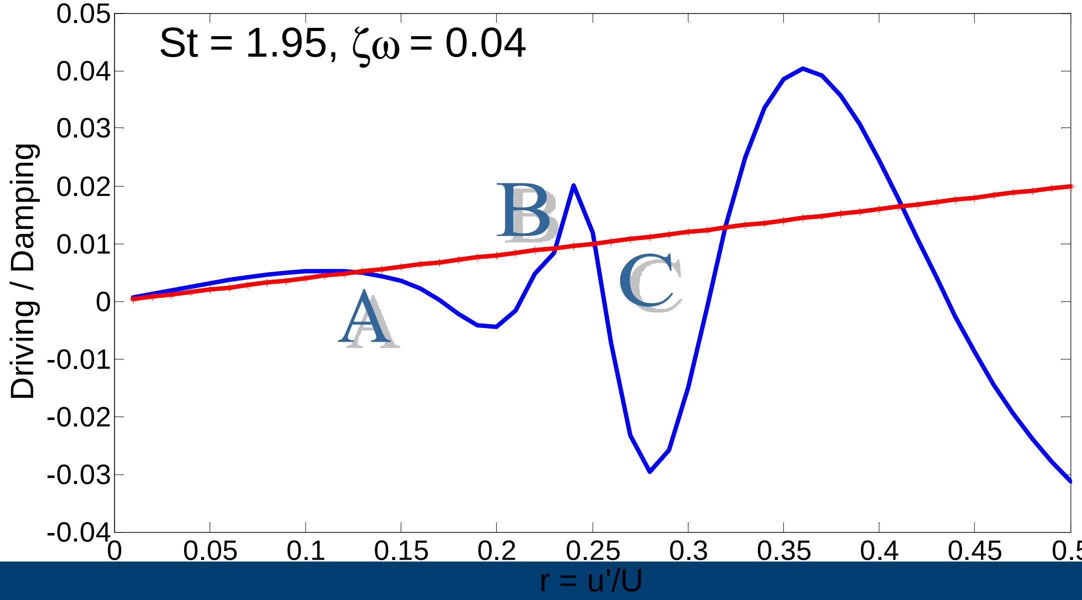
The higher harmonics in the heat release signal do not affect the stability of a periodic solution!

By definition, the integral of dE/dt goes to zero for a periodic solution. Whether these periodic solutions are stable or unstable can be identified by examining the change in sign of the above integral.

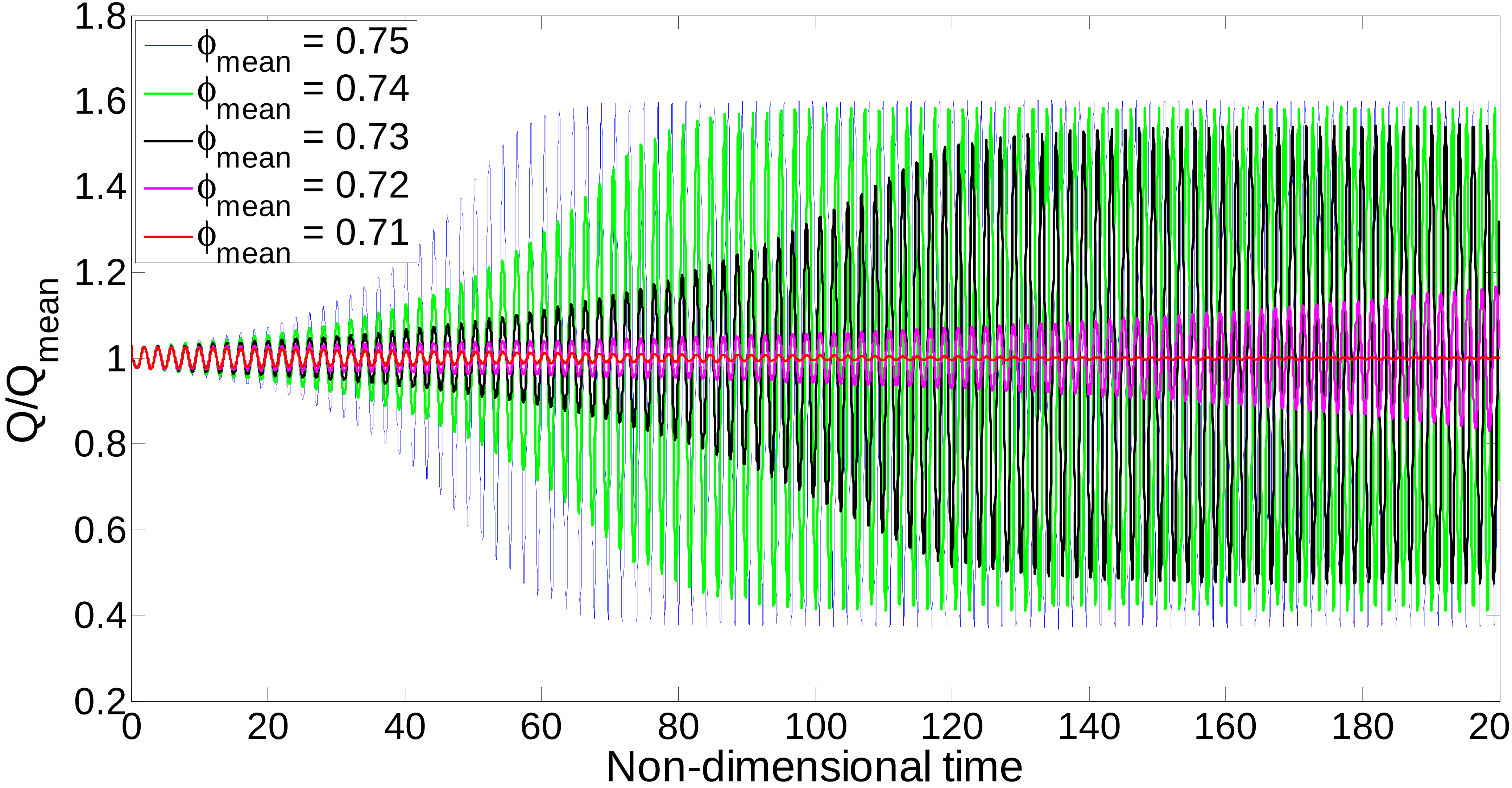
The premixed flame is susceptible to triggering
Both subcritical and supercritical bifurcations are possible







Self-excited instabilities



Future work

- Use M.P.Juniper's flow instability tool to predict the wave-speeds of convected disturbances, which is an input to the flame model
- Map out the bifurcation diagram as a function of various parameters (length of tube, flame position, damping factor and flame parameters such as mean equivalence ratio)
- Extend the above analyses to more realistic flame geometries and to stratified flames