On the formation of pancake eddies in stratified turbulence

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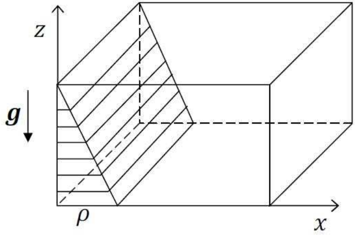
Background

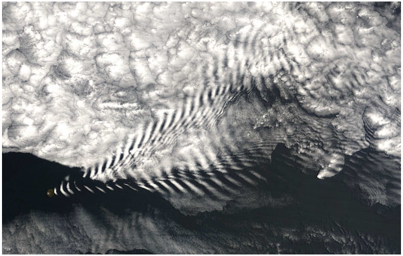
Turbulence in a fluid with a **stable density gradient** can be observed in:

- atmosphere $\longrightarrow$ variations in T with height
- oceans $\longrightarrow$ variations in T and S with depth



$N = \sqrt{-\frac{g}{\rho_0}\frac{d\rho}{dz}}$: Brunt-Väisälä frequency

Motivation

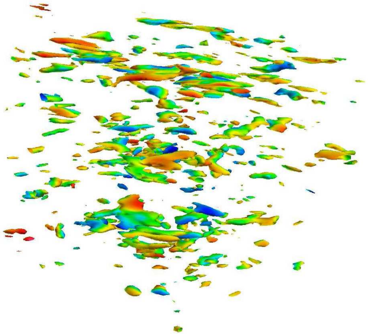


Internal gravity waves in the wake of the Ile d'Amsterdam in the Indian Ocean

Eruption of the Cleveland Volcano in the Aleutian Islands of Alaska

Stably stratified turbulence

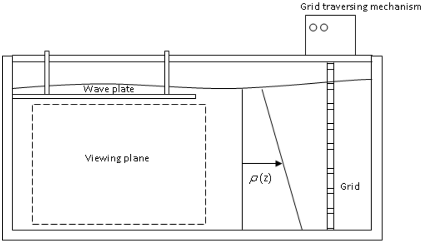
Stratified turbulence is anisotropic; it contains flat **pancake eddies**



Enstrophy contours at the end of DNS of decaying stratified turbulence [1]

[1] L. Liechtenstein, *PhD Thesis*, Ecole Centrale de Lyon, 2005

Inhomogenous turbulence experiments

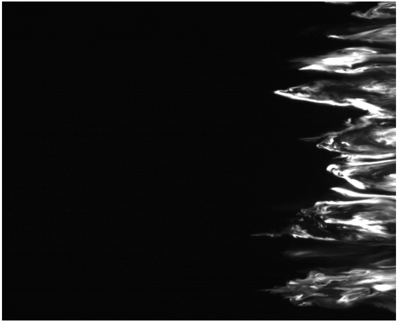


Experimental setup

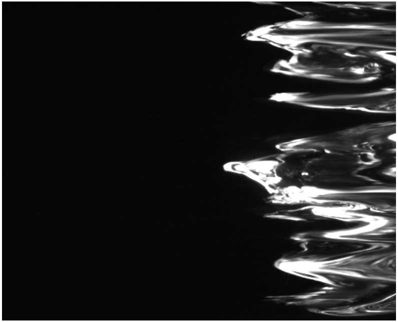
- Salt-stratified solution
- Grid: rake of vertical bars
- Rake is traversed in right half of tank only
- Visualisation of turbulent/quiescent interface: **turbulent front**
- Pearlessence → wave motion
- Fluorescent dye → fluid motion

Pearlessence experiments

Fluorescent dye experiments



$Nt = 37.1$



$Nt = 79.9$

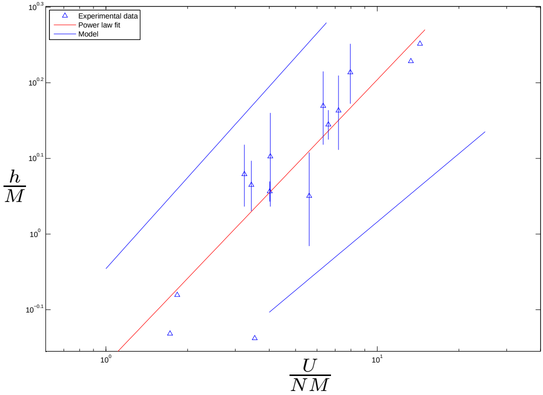
Conclusion: fluid intrusions grow out of the turbulent cloud and radiate off horizontally-travelling wave packets

Semi-empirical model

- Assume turbulence behaves like non-stratified turbulence up to "collapse" of turbulent cloud & pancake formation
- Empirical law: $\left(\frac{u}{U}\right)^2 = A\left(\frac{U(t-t_0)}{M}\right)^{-n}$, $n = 1.2 - 1.4$ [2]
- $\frac{du^2}{dt} = -\alpha\frac{u^3}{L}$
- Therefore: $L = -\alpha\frac{u^3}{du^2/dt} = B(Ut)^{1-n/2}M^{n/2}$
- Froude number: $Fr = \frac{u}{NL} = \left(\frac{A^{1/2}}{B}\right)\frac{1}{Nt}$
- Collapse occurs when $Fr \sim 1 \implies t_c \sim N^{-1}$
- KEY ASSUMPTION: $h \sim L_c$
- Normalized pancake height: $\frac{h}{M} \sim \left(\frac{U}{NM}\right)^m$, $m = 0.3 - 0.4$

[2]: G. Comte-Bellot & S. Corrsin, *Journal of Fluid Mechanics*, 25(04):657-682, 1966

Experimental results vs. model



Normalized **pancake height** as a function of grid Froude number

Direct Numerical Simulations

Solve the equations of motion for a linearly stratified fluid with the Boussinesq approximation (buoyancy $b = \frac{\rho' g}{\rho_0}$, vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, modified pressure $P = \frac{p'}{\rho_0} + \frac{1}{2}u^2$)

- $\nabla \cdot \mathbf{u} = 0$ (mass)
- $\frac{\partial \mathbf{u}}{\partial t} = \mathbf{u} \times \boldsymbol{\omega} - \nabla P + \nu \nabla^2 \mathbf{u} - b\mathbf{e_z}$ (momentum)
- $\frac{\partial b}{\partial t} = -\mathbf{u} \cdot \nabla b + D\nabla^2 b + N^2 u_z$ (energy)

Pseudospectral method:

- Fourier-transform the equations of motion
- Evaluate non-linear terms $\mathbf{u} \times \boldsymbol{\omega}$ and $-\mathbf{u} \cdot \nabla b$ in real space
- Numerical time integration using a 3rd order Runge-Kutta method

Numerical results: velocity magnitude contours

Numerical results: PV contours

Conclusions

- In inhomogeneous stratified turbulence pancake formation occurs through the growth of intrusions out of the turbulent cloud
- Pancakes/Intrusions radiate off horizontal waves as they grow
- Semi-empirical model predicts a power-law dependence of the pancake height on the initial Froude number
- The experimental curve falls within the bounds predicted by the model
- Suggests that **turbulent mixing** at the integral lengthscale level is important in pancake formation
- Can this mechanism be transposed to homogeneous turbulence?