UNIVERSITY OF CAMBRIDGE

Sensitivity Analysis of Spiral Vortex Breakdown

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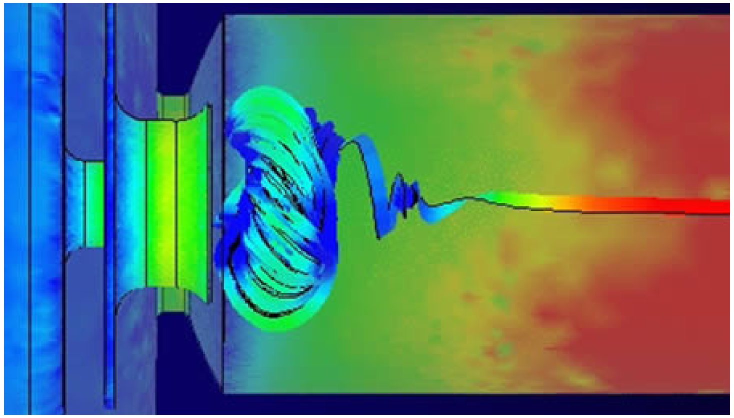
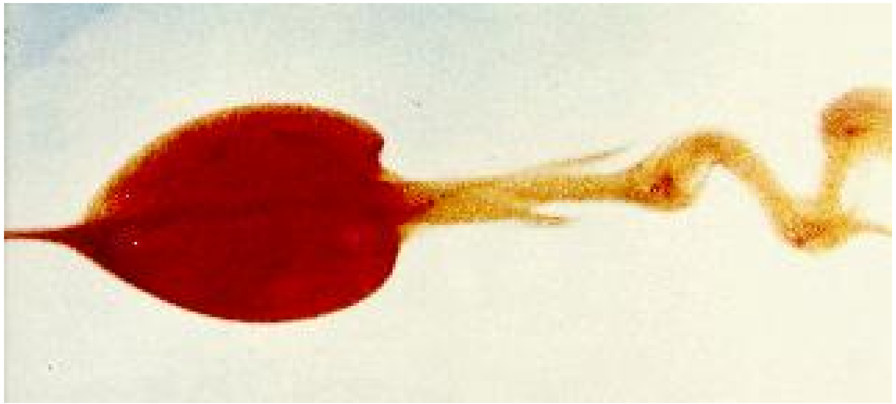
We are studying the laminar vortex breakdown bubble because it is a toy model for the recirculating zone in a gas turbine combustion chamber.

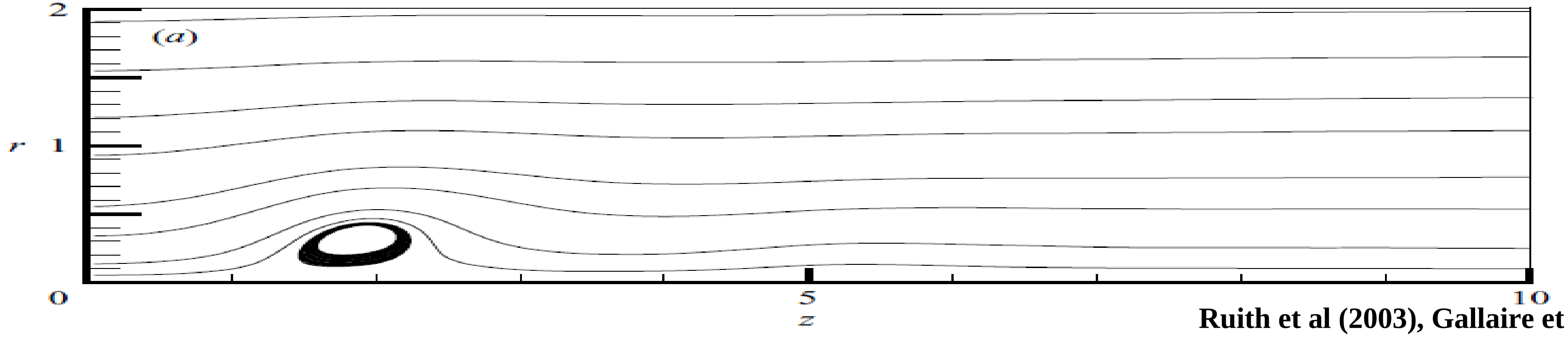
Gas turbine combustion (Re ~ 10^6)



Vortex breakdown bubble (Re ~ 200)



Vortex breakdown bubble (Sw=1.0, Re = 200)

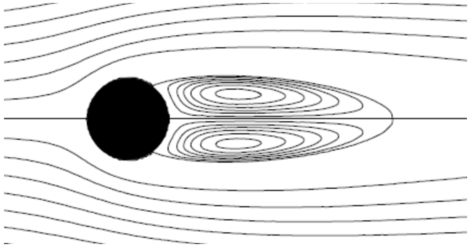


Ruith et al (2003), Gallaire et al (2006)

We use techniques that have been used to study vortex shedding behind a cylinder.

Giannetti & Luchini, JFM (2007) and Hill AIAA (1992)

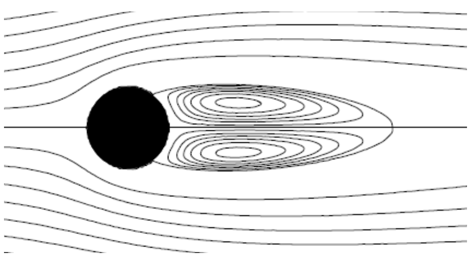
Base flow

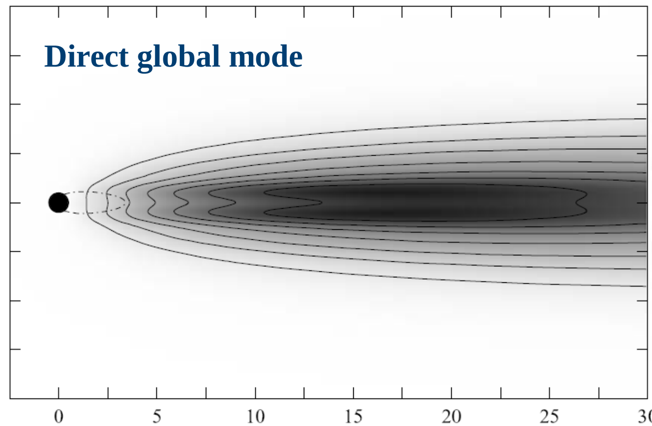


Solve Navier-Stokes equations to obtain a steady, axisymmetric solution.

We use techniques that have been used to study vortex shedding behind a cylinder.

Giannetti & Luchini, JFM (2007) and Hill AIAA (1992)





Superpose small perturbations onto steady base flow.

Solve 2D eigenvalue problem.

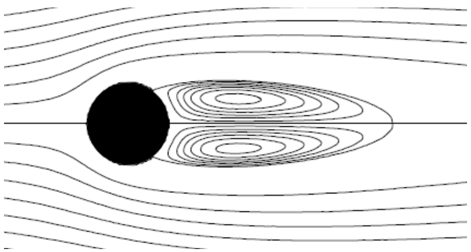
$$\frac{\mathrm{dq}}{\mathrm{d}t} = \mathbf{L}\mathrm{q},$$

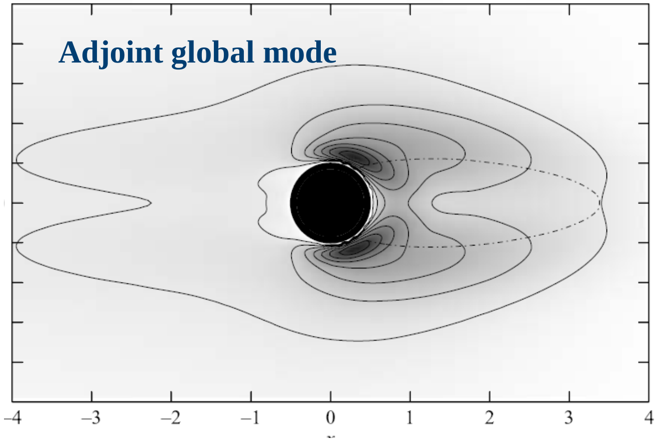
$$\mathrm{q}(x, r, \theta, t) = \hat{\mathrm{q}}(x, r) e^{\mathrm{i}m\theta + \lambda t}, \quad \lambda \equiv \sigma + \mathrm{i}\omega$$

$$\lambda \hat{\mathrm{q}} = \mathbf{L}_m \hat{\mathrm{q}}.$$

We use techniques that have been used to study vortex shedding behind a cylinder.

Giannetti & Luchini, JFM (2007) and Hill AIAA (1992)





Derive adjoint (conjugate transpose) equations.

Solve another 2D eigenvalue problem.

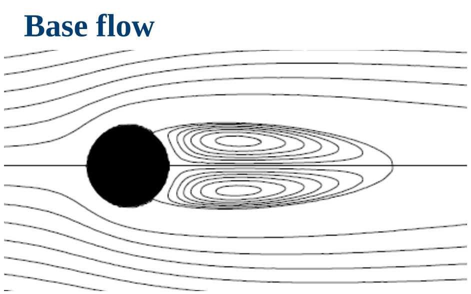
$$-\frac{\mathrm{d}\mathbf{q}^+}{\mathrm{d}t} = \mathbf{L}^+\mathbf{q}^+$$

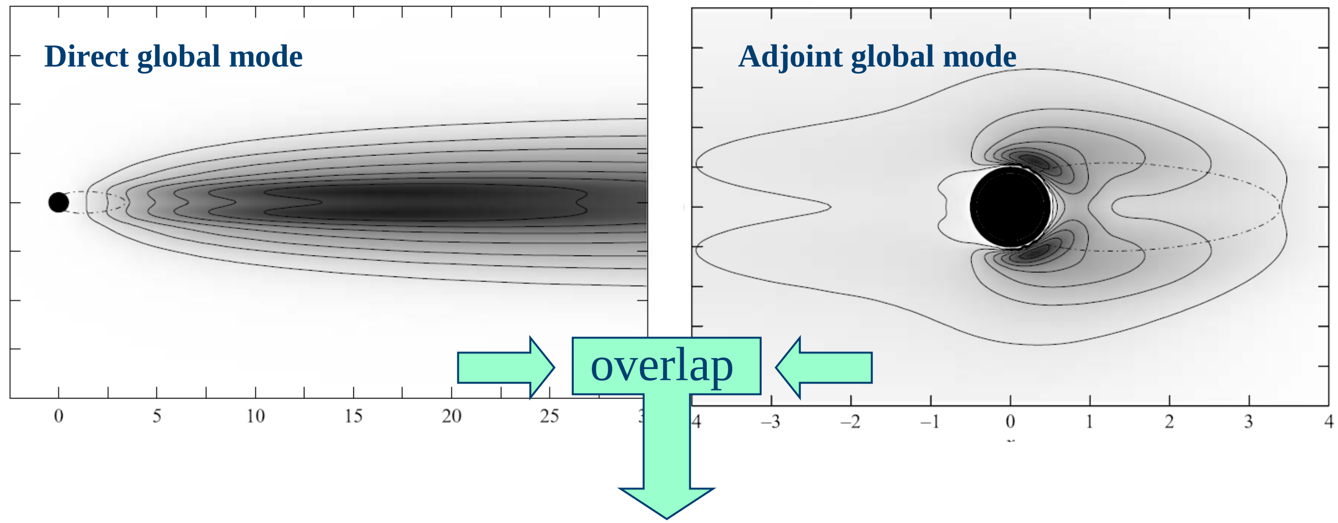
$$\mathbf{q}^+(x,r,\theta,t) = \hat{\mathbf{q}}^+(x,r)e^{\mathrm{i}m\theta-\lambda^* t}$$

$$\lambda^*\hat{\mathbf{q}}^+ = \tilde{\mathbf{L}}_m^+\hat{\mathbf{q}}^+$$

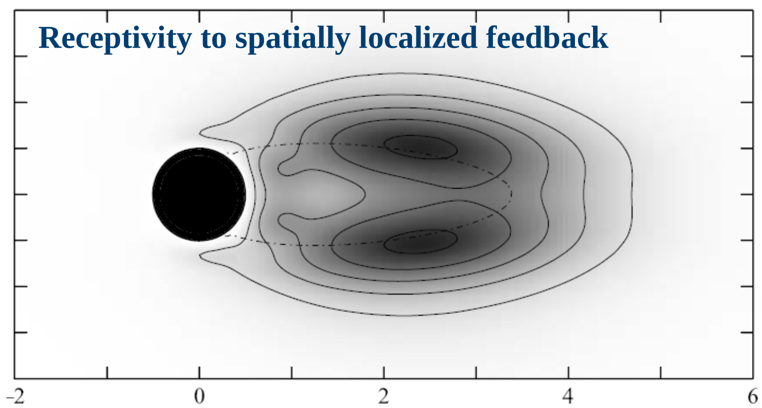
We use techniques that have been used to study vortex shedding behind a cylinder.

Giannetti & Luchini, JFM (2007) and Hill AIAA (1992)



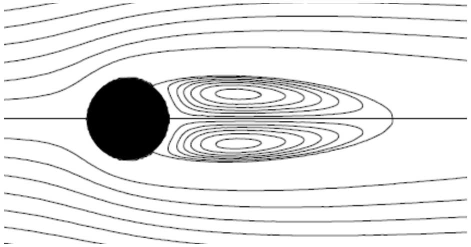


$$|\nabla_G \lambda|_{max} = \frac{|\hat{\mathrm{m}}^H||\hat{\mathrm{m}}^+|}{\langle \hat{\mathrm{m}}^+, \hat{\mathrm{m}} \rangle}.$$



The absolute growth rate is a useful diagnostic result. It is obtained from a local stability analysis using INSTAFLOW.

Base flow

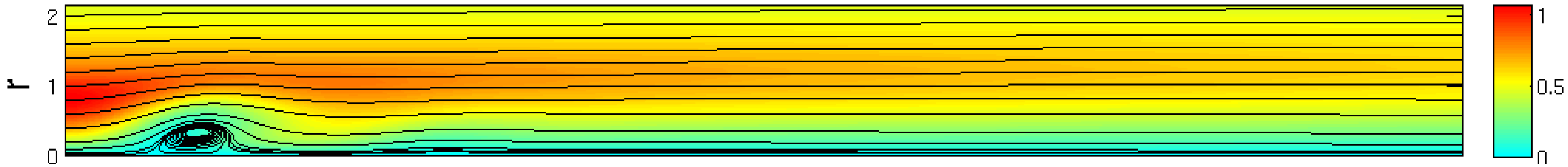


continuous direct LNS* → continuous direct O-S** → discretized direct O-S** → $\omega_0(X)$

Absolute growth rate

* LNS = Linearized Navier-Stokes equations
** O-S = Orr-Sommerfeld equation

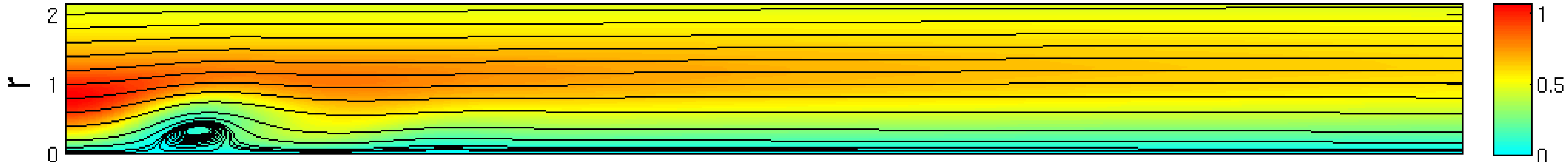
We checked the results for the linear direct global mode against previously-published results for the nonlinear global mode.



Direct eigenvalues

Sw=1.0	Xmax	Rmax	Sx	Sr	Growth rate	Frequency
Fully 3D DNS (Ruith)	**20.0**	**10.0**	**193**	**61**	**0.0359**	**1.1800**
Local nonlinear analysis (Gallaire)	**-**	**-**	**-**	**-**	**-**	**1.2200**
M1	**20.0**	**8.0**	**513**	**127**	**0.035214**	**1.165476**

We checked the effect of the grid resolution and size of the domain.



Direct eigenvalues

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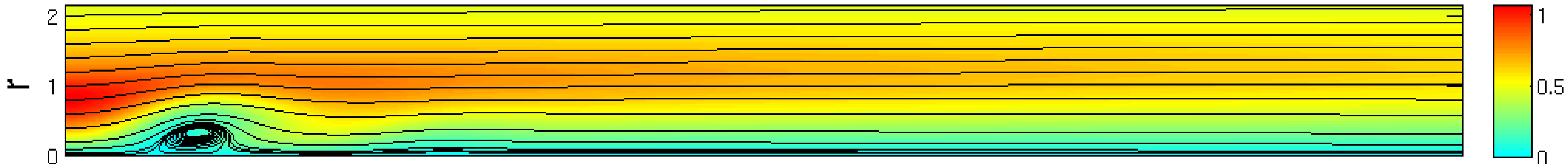
Local nonlinear analysis (Gallaire)	-	-	-	-	-	1.2200
M1	20.0	8.0	513	127	0.035214	1.165476
M2	20.0	8.0	257	127	0.035177	1.165453
M3	25.4	10.4	257	127	0.034348	1.162470

We checked the results for the adjoint global mode against the direct global mode.



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M3	**25.4**	**10.4**	**257**	**127**	**0.034348**	**1.162470**

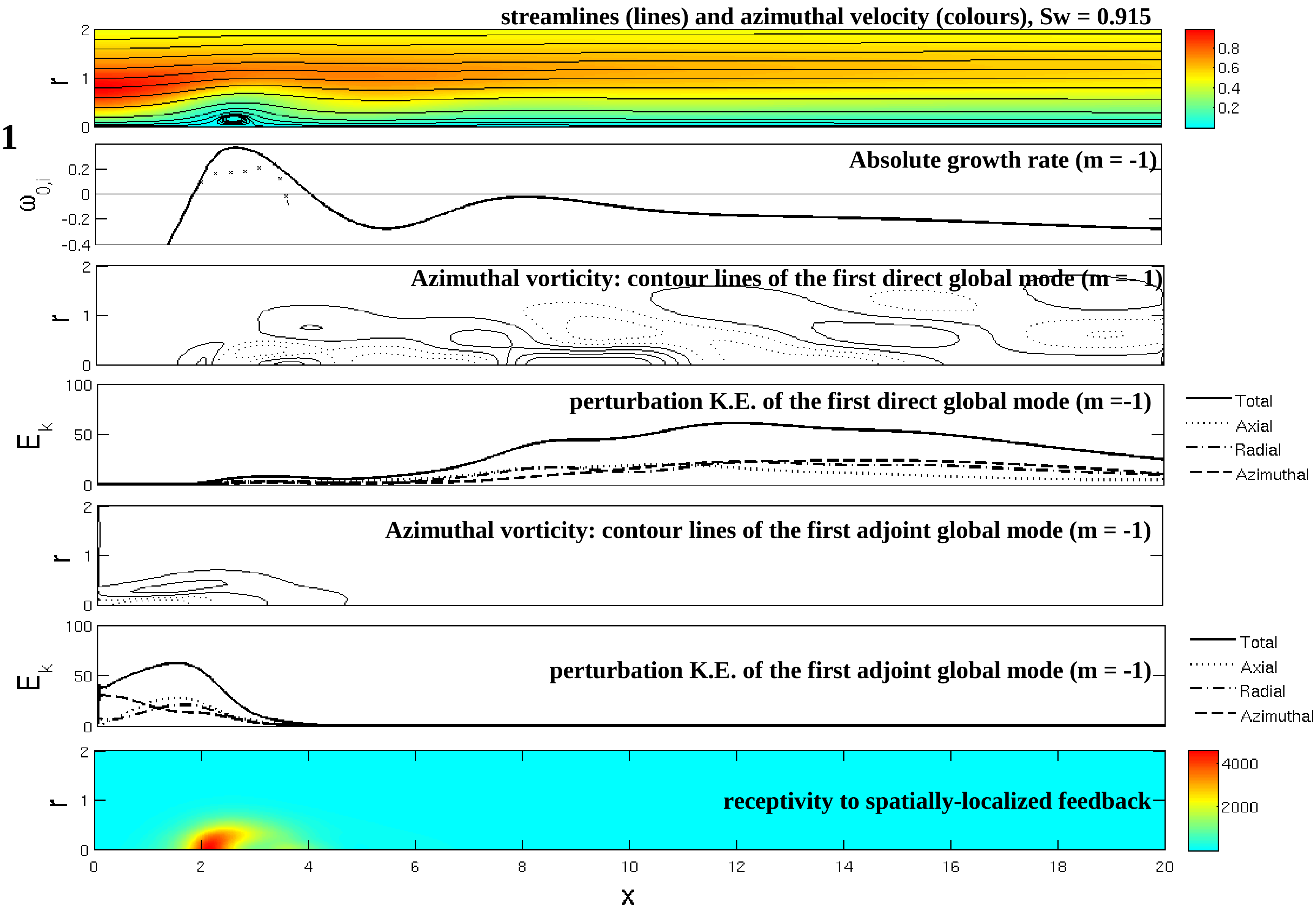
Adjoint eigenvalues

	Growth rate	Frequency	Absolute discrepancy	Relative discrepancy (%)
M1	**0.037663**	**-1.165434**	**0.002449**	**0.210064**

At Sw=0.915, the flow is just globally unstable for m=-1. The recirculation bubble is absolutely unstable. The overlap of direct and adjoint modes shows that the wavemaker region is in the recirculation bubble.

Sw = 0.915

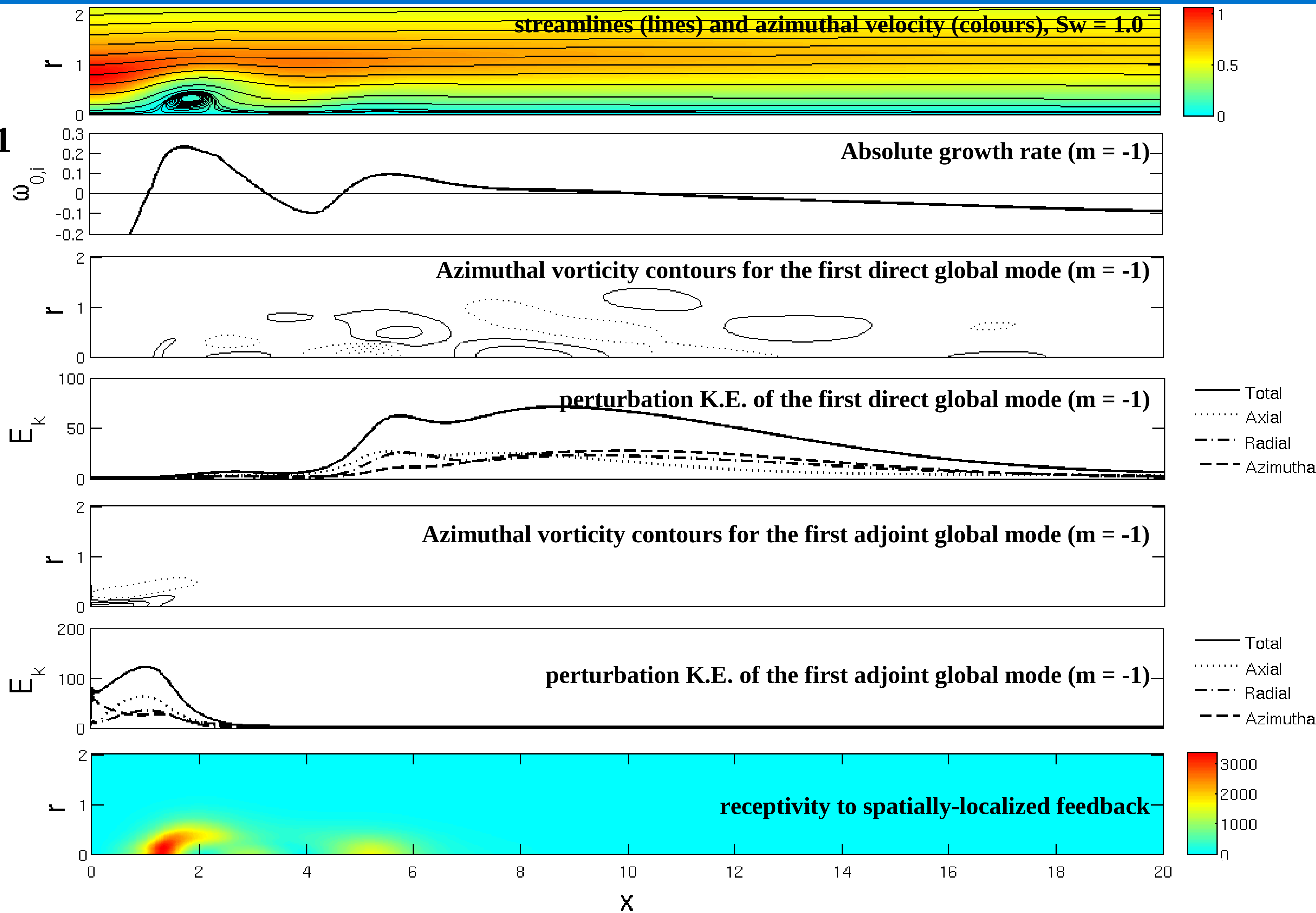
m = -1



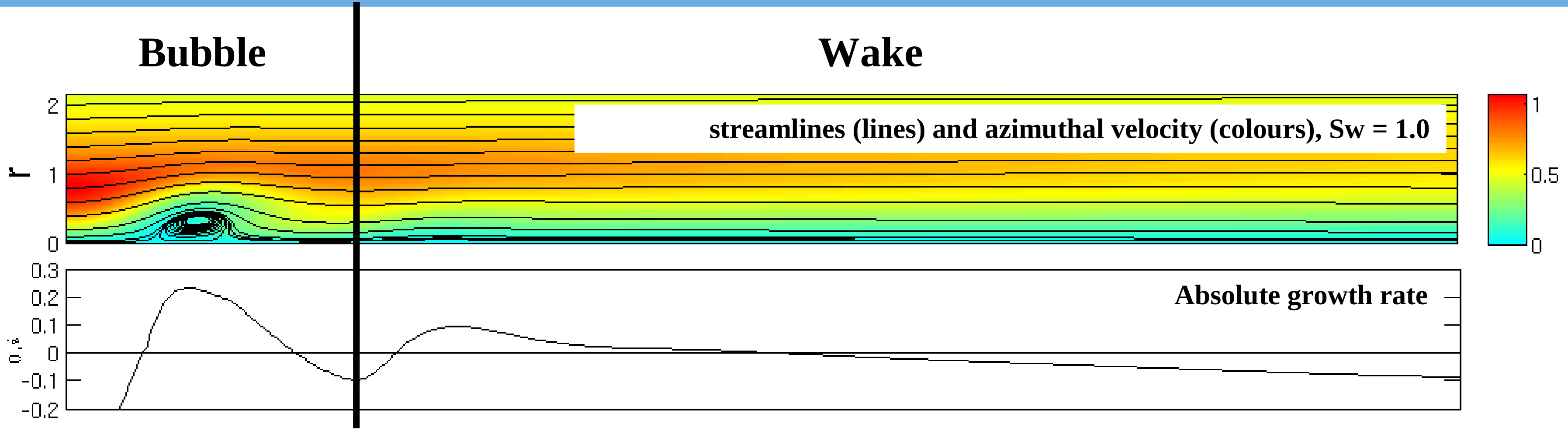
At Sw=1.0, there are two regions of absolute instability. The overlap of direct and adjoint modes shows that the wavemaker region is still predominantly in the recirculation bubble.

Sw = 1.00

m = -1



We calculated global modes of the bubble and wake separately. The frequency of the global mode seems to be driven by the bubble but the growth rate seems to be enhanced by the nearby marginally-stable wake. It is like a coupled oscillator.

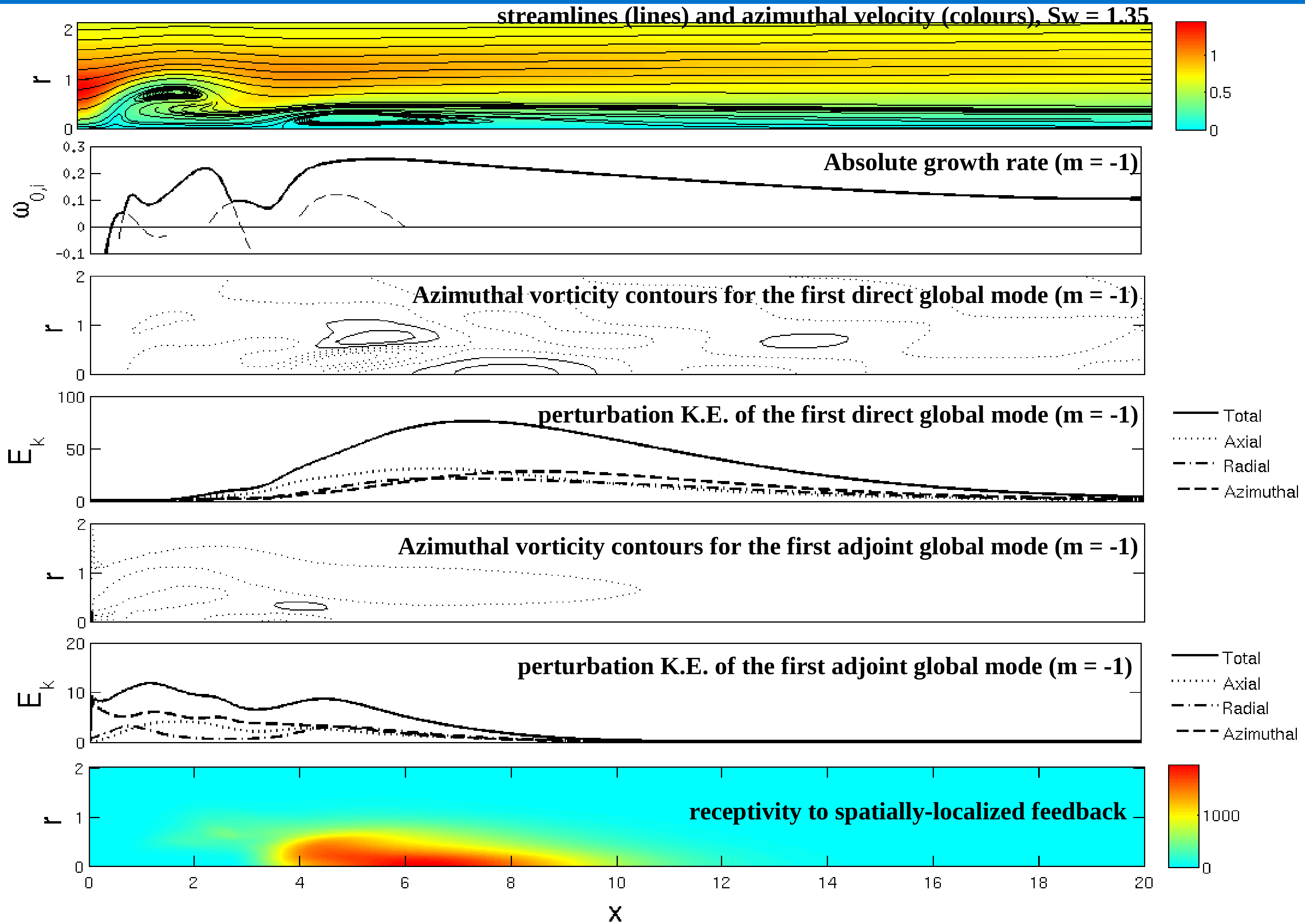


	Growth rate	Frequency
Complete global	0.0352	1.1655
Bubble	0.0132	1.1698
Wake	-0.0475	1.0852

As the swirl increases, a second recirculation bubble forms in the wake. The wavemaker region moves into the wake bubble.

Sw = 1.35

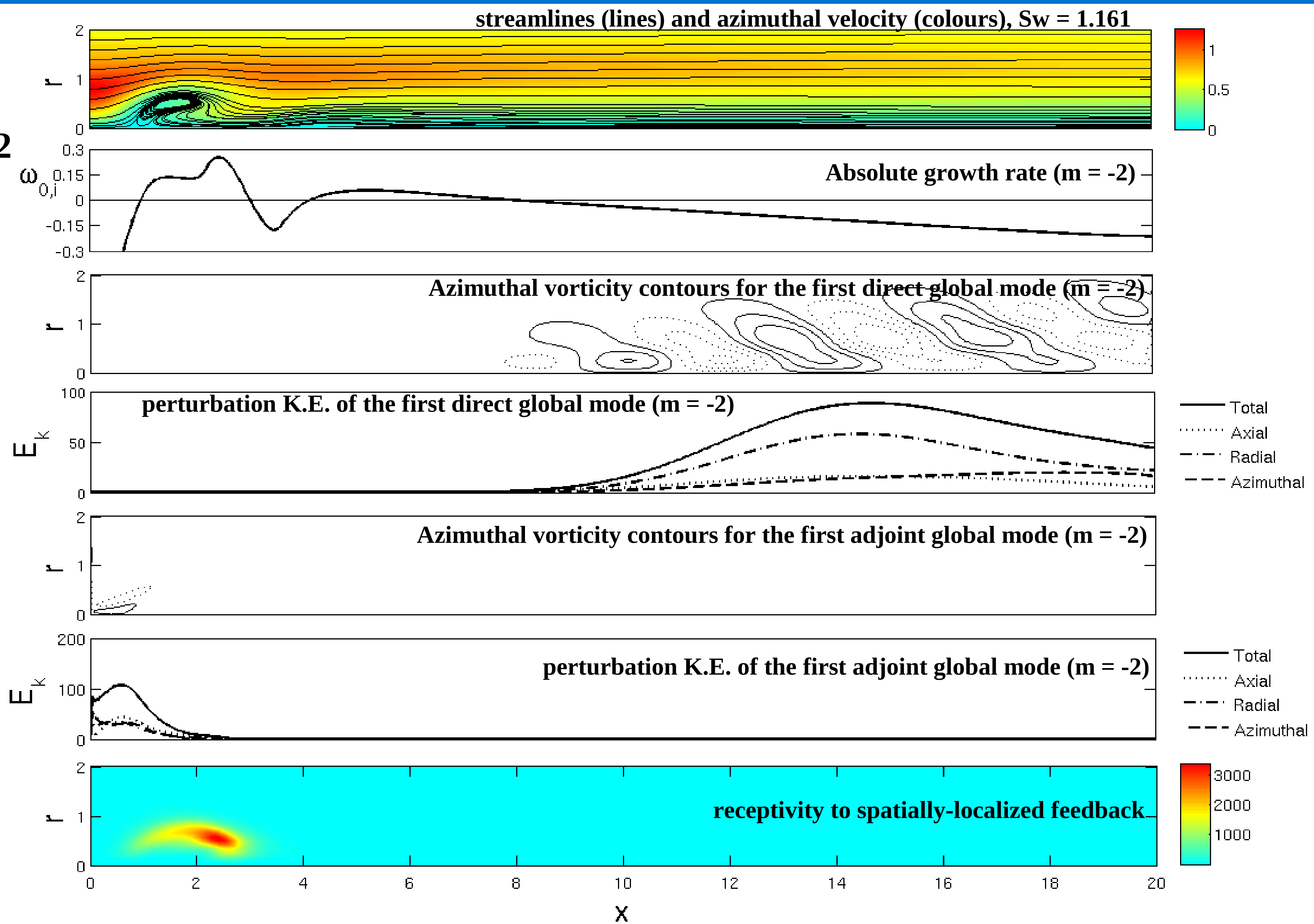
m = -1



The m = -2 mode becomes unstable at Swirl = 1.161. The wavemaker region is in the recirculation bubble.

Sw = 1.161

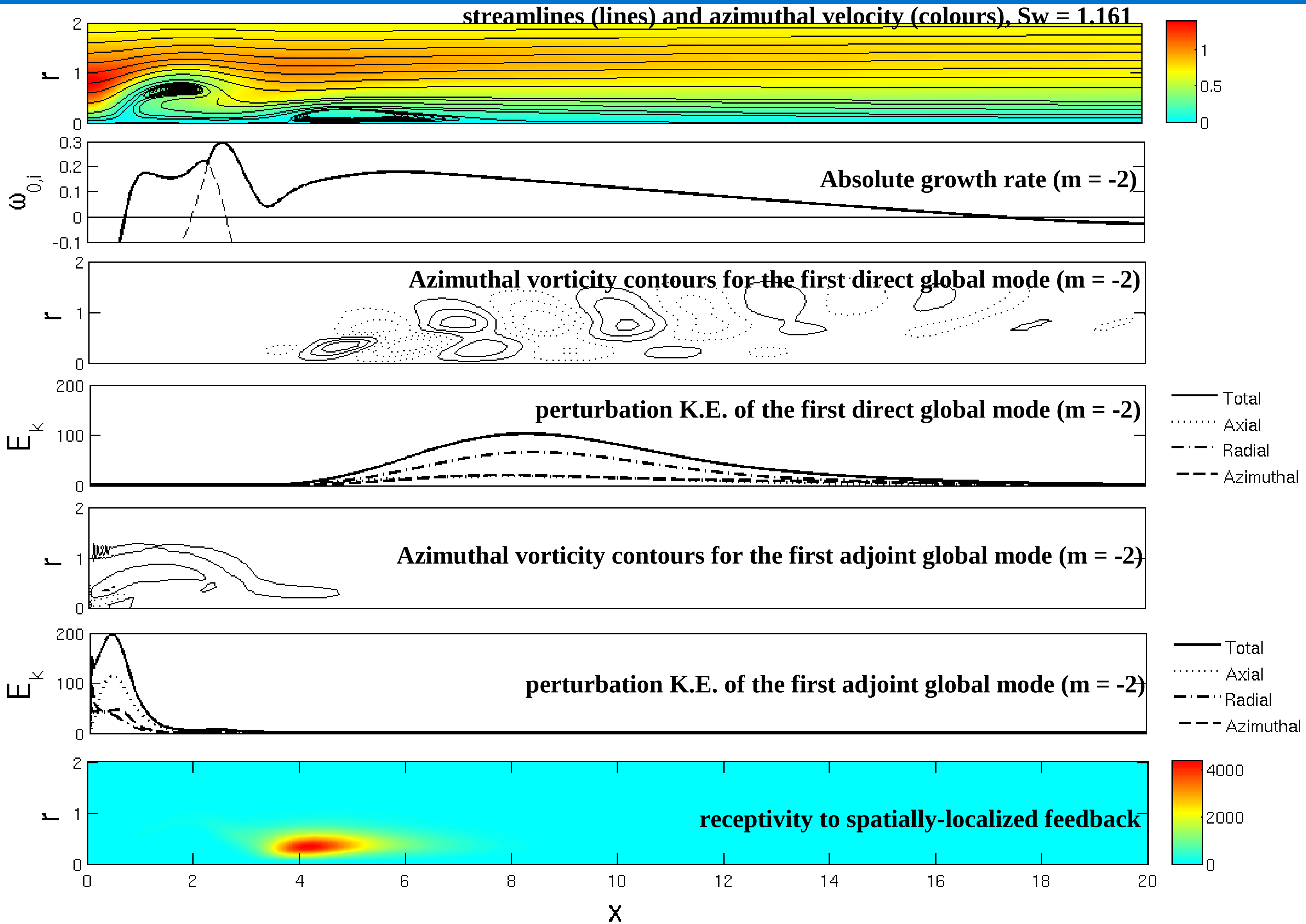
m = -2



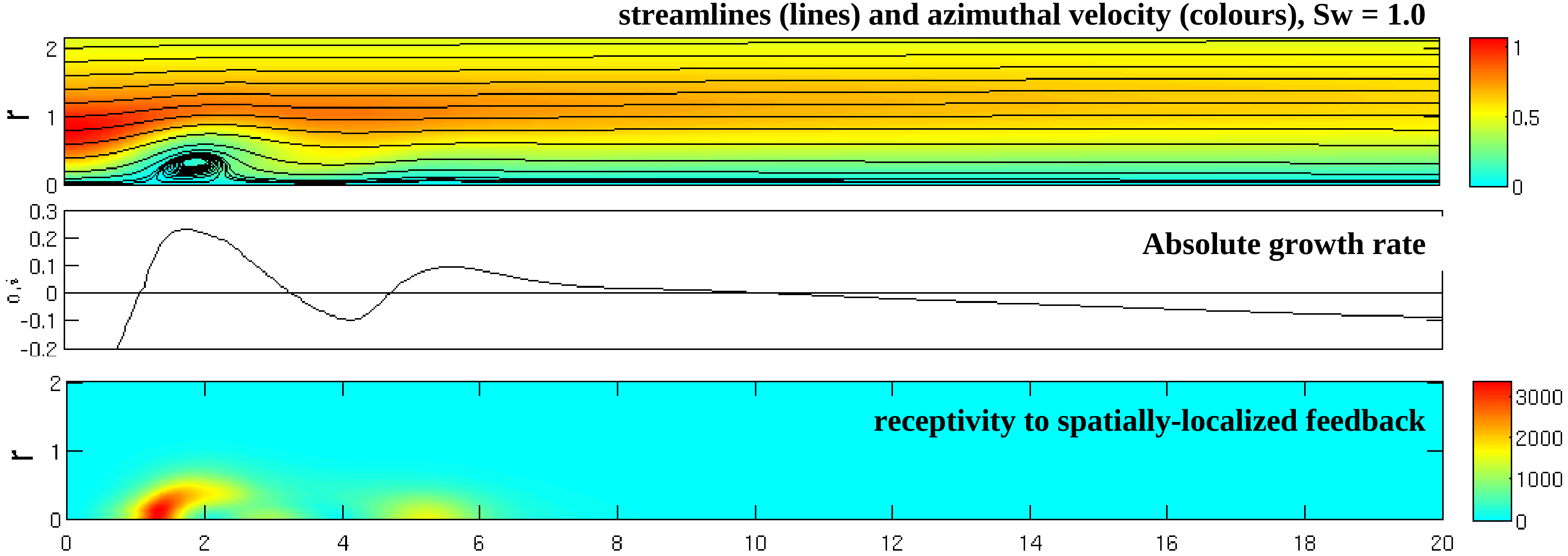
As the swirl increases, the wavemaker region moves into the recirculating wake, as for the m = -1 mode.

Sw = 1.3

m = -2



1. At low swirls, the wavemaker region is in the bubble, not the wake.



2. As the swirl increases, the wavemaker region moves into the wake.

3. These effects are seen for both the m = -1 and m = -2 modes.

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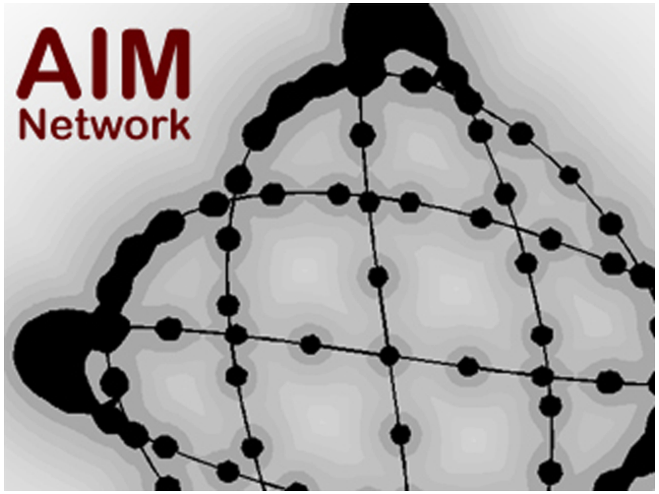

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Saturday 27th August 2011,

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